Foundations of Algebra

One Team One Goal Student Success

Foundations of Algebra Teaching & Learning Framework

Semester 1

Module 1 2 weeks	Module 2 2 weeks	Module 3 2 weeks	Module 4 6 weeks	Module 5 6 weeks
Number Sense & Quantity	**Arithmetic to Algebra**	**Proportional Reasoning**	**Equations & Inequalities**	**Quantitative Reasoning with Functions**
MFANSQ1 Relationships (4 operations; multiples of fractions; multiple/divide by powers of ten with decimals; compare fractions/decimals) **MFANSQ2 +/-** (Real number+- on a number line & the meaning of zero) **MFANSQ3 Irrationals** (Irrational number approximations; adding & multiplying with rational & irrational numbers) **MFANSQ4 Computation** (Compute multi-digit decimals; compute with rational numbers; division of fractions by fractions; multi-step problems with any form of rational number) **Algebra I Unit 1:** **MGSE9-12.N.RN.3** (Properties of rational & irrational numbers)	**MFAAA1 Equivalent Expressions** (Commutative & distributive properties; numerical & algebraic expressions; add, subtract & multiply algebraic expressions; equivalent expressions; evaluate formulas) **MFAAA2 Exponents** (integer exponents; formulas; square & cube roots; Pythagorean Theorem) **Algebra I Unit 1:** **MGSE9-12.A.SSE.1** (Interpret expressions in context) **MGSE9-12.A.SSE.1a-b** (Interpret formulas & expressions in context) **MGSE9-12.A.APR.1** (Add, subtract & multiply polynomials) **MGSE9-12.N.RN.2** (Expressions with Radicals)	**MFAPR1 Equivalent Ratios** (Equivalent ratios) **MFAPR2 Proportions** (Fraction equivalence & division; percent problems) **MFAPR3 Graphing** (Unit rates as slope; similar triangles and slope; compare proportions in multiple representations) **Algebra I Unit 1:** **MGSE9-12.N.Q.1-3** (Reason quantitatively & use units to solve problems)	**MFAEI1 One Variable** (Solve equations & inequalities & justify solutions) **MFAEI2 Units** (Scale, units, graphing) **MFAEI3 Two Variables** (Algebraic models; graphing calculators; systems of equations) **MFAEI4 Literal Equations** (Solve for a specific variable) **Algebra I Unit 1:** **MGSE9-12.N.Q.1-3** (Reason quantitatively & use units to solve problems) **Algebra I Unit 2:** **MGSE9-12.A.CED.1-4** (Create equations that describe numbers or relationships) **MGSE9-12.A.REI.1,3,5** (Solve equations & inequalities 1-2 variable) **MGSE9-12.A.REI.6** (Solve systems) **MGSE9-12.A.REI.10-12** (Solve equations & inequalities 2 variables)	**MFAQR1 Characteristics** (Domain & range) **MFAQR2 Compare & Graph** (Rates of change; linear & non-linear; key features; compare with multiple representations) **MFAQR3 Construct & Interpret** (Write; variables in context; function notation) **Algebra I Unit 2:** **MGSE9-12.F.BF.1** (Write a function) **MGSE9-12.F.BF.1a,2** (Arithmetic sequences) **MGSE9-12.F.IF.1** (Input vs. output) **MGSE9-12.F.IF.2** (Function notation) **MGSE9-12.F.IF.3-4** (Sequences & characteristics) **MGSE9-12.F.IF.5-6** (Rate of change) **MGSE9-12.F.IF.7,7a,9** (Analyze functions)

These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.
All units will include the Mathematical Practices and indicate skills to maintain

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

COBB TEACHING AND LEARNING STANDARDS

FA Algebra I Teaching & Learning Framework

Semester 2			
Unit 3 6 weeks	**Unit 4** 4 weeks	**Unit 5** 3 weeks	**Unit 6** 5 weeks
Modeling & Analyzing Quadratic Functions	**Modeling & Analyzing Exponential Functions**	**Comparing & Contrasting Functions**	**Describing Data Review and Extend**
MGSE9-12.A.SSE.2 (Interpret the structure of expressions) **MGSE9-12.A.SSE.3,3a-b** (Equivalent forms of expressions) **MGSE9-12.A.CED.1-2,4** (Create equations that describe numbers or relationships) **MGSE9-12.A.REI.1** (Justify how to solve an equation) **MGSE9-12.A.REI.4,4a-b** (Methods of solving quadratics) **MGSE9-12.F.BF.1,3** (Write a function & build new functions) **MGSE9-12.F.IF.1** (Input vs. output) **MGSE9-12.F.IF.2** (Function notation) **MGSE9-12.F.IF.4** (Characteristics) **MGSE9-12.F.IF.5-6** (Rate of change) **MGSE9-12.F.IF.7,7a** (Graph functions) **MGSE9-12.F.IF.8** (Write a function) **MGSE9-12.F.IF.8a,9** (Compare & contrast functions)	**MGSE9-12.A.CED.1-2** (Create equations 1-2 variables) **MGSE9-12.A.REI.1** (Justify how to solve an equation) **MGSE9-12.F.BF.1** (Write a function) **MGSE9-12.F.BF.1a,2** (Arithmetic & geometric sequences) **MGSE9-12.F.BF.3** (Build new functions) **MGSE9-12.F.IF.1** (Input vs. output) **MGSE9-12.F.IF.2** (Function notation) **MGSE9-12.F.IF.3-4** (Sequences & characteristics) **MGSE9-12.F.IF.5-6** (Rate of change) **MGSE9-12.F.IF.7,7e** (Graph functions) **MGSE9-12.F.IF.9** (Compare functions)	**MGSE9-12.F.LE.1** (Linear vs exponential) **MGSE9-12.F.LE.1a** (Growth of functions) **MGSE9-12.F.LE.1b,c,2-3** (Changes in rate and relating to context) **MGSE9-12.F.LE.5** (Interpret parameters) **MGSE9-12.F.BF.3** (Build new functions) **MGSE9-12.F.IF.1** (Input vs. output) **MGSE9-12.F.IF.2** (Function notation) **MGSE9-12.F.IF.4** (Characteristics) **MGSE9-12.F.IF.5-6** (Rate of change) **MGSE9-12.F.IF.7** (Graph functions) **MGSE9-12.F.IF.9** (Compare functions)	**MGSE9-12.S.ID.1** (Dot plots, histograms & box plots) **MGSE9-12.S.ID.2** (Compare data distribution) **MGSE9-12.S.ID.3** (Shape, center & spread) **MGSE9-12.S.ID.5-6** (Bivariate data) **MGSE9-12.S.ID.6a,c** (Function of best fit) **MGSE9-12.S.ID.7-9** (Slope, correlation coefficient, causation & correlation) **Review: All standards by differentiating for student needs** **Extend:** **MGSE9-12.G.CO.1** (Precise definitions)
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Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities

Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see as being composed of the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning.
High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Number Sense and Quantity

Students will compare different representations of numbers (i.e. fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFANSQ1. Students will analyze number relationships.

    a. Solve multi-step real world problems, analyzing the relationships between all four operations. *For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each.* (MGSE3.OA.6, MGSE4.OA.3)

    b. Understand a fraction a/b as a multiple of 1/b. (MGSE4.NF.4)

    c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MGSE5.NBT.2)

    d. Compare fractions and decimals to the thousandths place. *For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value.* (MGSE4.NF.2; MGSE5. NBT.3, 4)

MFANSQ2. Students will conceptualize positive and negative numbers (including decimals and fractions.)

    a. Explain the meaning of zero. (MGSE6.NS.5)

    b. Represent numbers on a number line. (MGSE6.NS.5, 6)

    c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFANSQ3. Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.

    a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. *For example, for $\sqrt{\ }$, show that $\sqrt{\ }$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations.* (MGSE8.NS.1, 2)

    b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFANSQ4. Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.

    a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)

    b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1, 2)

    c. Interpret and solve contextual problems involving division of fractions by fractions. *For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt?* (MGSE6.NS.1)

    d. Illustrate and explain calculations using models and line diagrams. (MGSE7. NS.1, 2)

    e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.E.3, MGSE9-12.N.Q.

Arithmetic to Algebra

Students will extend arithmetic operations to algebraic modeling.

MFAAA1. Students will generate and interpret equivalent numeric and algebraic expressions.

   a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
   b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
   c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
   d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)
   e. Generate equivalent expressions using properties of operations and understand various representations within context. *For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times".* (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2;MGSE9-12.A.SSE.3)
   f. Evaluate formulas at specific values for variables. *For example, use formulas such as A = l x w and find the area given the values for the length and width.* (MGSE6.EE.2)

MFAAA2. Students will interpret and use the properties of exponents.

   a. Substitute numeric values into formulas containing exponents, interpreting units consistently.
   b. Use properties of integer exponents to find equivalent numerical expressions. *For example, 32 x 3 -5 = 3 -3 = = .* (MGSE8.EE.1)
   c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2)
   d. Use square root and cube root symbols to represent solutions to equations of the form $x2 = p$ and $x3 = p$, where p is a positive rational number. (MGSE8.EE.2)
   e. Use the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given two legs). (MGSE8.G.7)

Proportional Reasoning

Students will use ratios to solve real-world and mathematical problems.

MFAPR1. Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations*. (MGSE6.RP.3)

MFAPR2. Students will recognize and represent proportional relationships between quantities**.**
   a. Relate proportionality to fraction equivalence and division. *For example, is equal to because both yield a quotient of ½ and, in both cases, the denominator is double the value the value of the numerator.* (MGSE4.NF.1)
   b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1, 2, 3; MGSE7.RP.1, 2)
   c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2, 3)

MFAPR3. Students will graph proportional relationships.
   a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
   b. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
   c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Equations and Inequalities

Students will solve, interpret, and create linear models using equations and inequalities.

MFAEI1. Students will create and solve equations and inequalities in one variable.
   a. Use variables to represent an unknown number in a specified set. (MGSE.6.EE2, 5, 6)
   b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
   c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)
   d. Represent and find solutions graphically.
   e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7, MGSE7.EE.4)

MFAEI2. Students will use units as a way to understand problems and guide the solutions of multi-step problems.

   a. Choose and interpret units in formulas. (MGSE9-12.N.Q.1)

   b. Choose and interpret graphs and data displays, including the scale and comparisons of data. (MGSE3.MD.3, MGSE9-12.N.Q.1)

   c. Graph points in all four quadrants of the coordinate plane. (MGSE6.NS.8)

MFAEI3. Students will create algebraic models in two variables.

   a. Create an algebraic model from a context using two-variable equations. (MGSE6.EE.6,8; MGSE8.EE.8; MGSE9-12.A.CED.2)

   b. Find approximate solutions using technology to graph, construct tables of values, and find successive approximations. (MGSE9-12.A.REI.10, 11)

   c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE.5; MGSE7.EE3; MGSE8.EE.8; CC9-12.A.CED.2)

   d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5, 6, MGSE7.EE4)

MFAEI4. Students will solve literal equations**.**

   a. Solve for any variable in a multi-variable equation. (MGSE6.EE.9, MGSE9 -12.A.REI.3)

   b. Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base in A = ½ bh*. (MGSE9-12.A.CED.4)

Quantitative Reasoning with Functions

Students will create function statements and analyze relationships among pairs of variables using graphs, table, and equations.

MFAQR1. Students will understand characteristics of functions.

   a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)

   b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)

   c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFAQR2. Students will compare and graph functions.

    a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2; MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)

    b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)

    c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)

    d. Use technology to graph non-linear functions. (MGSE8.F.3, MGSE9-12.F.IF.7)

    e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)

    f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.

    a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)

    b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)

    c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. (MGSE9-12.F.IF.2)

The Real Number System (N.RN)

Use properties of rational and irrational numbers.

MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

MGSE9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational

Quantities (N.Q.)

Reason quantitatively and use units to solve problems.

MGSE9-12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems: a. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs; b. Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor); c. Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

MGSE9-12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

MGSE9-12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. For example, money situations are generally reported to the nearest cent (hundredth). Also, an answers' precision is limited to the precision of the data given.

Seeing Structure in Expression (A.SSE)

Interpret the structure of expressions

MGSE9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

   a. Interpret parts of an expression, such as terms, factors, and coefficients, in context.

   b. Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.

MGSE9-12.A.SSE.2 Use the structure of an expression to rewrite it in different equivalent forms. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

MGSE9-12.A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

   a. Factor any quadratic expression to reveal the zeros of the function defined by the expression.

   b. Complete the square in a quadratic expression to reveal the maximum and minimum value of the function defined by the expression.

Arithmetic with Polynomials and Rational Expressions (A.APR)

Perform arithmetic operations on polynomials

MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.

Creating Equations (A.CED)
Create equations that describe numbers or relations

MGSE9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).

MGSE9-12.A.CED.2 Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase "in two or more variables" refers to formulas like the compound interest formula, in which A = A = $P(1 + r/n)^{nt}$ has multiple variables.

MGSE9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

MGSE9-12.A.CED.4 Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. Examples: Rearrange Ohm's law $V = IR$ to highlight resistance R; Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.

Reasoning with Equations and Inequalities (A.REI)

Understand solving equations as a process of reasoning and explain the reasoning

MGSE9-12.A.REI.1 Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

Solve equations and inequalities in one variable

MGSE9-12.A.REI.3 Solve linear equations and inequalities in one variable including equations with coefficients represented by letters. For example, given $ax + 3 = 7$, solve for x.

MGSE9-12.A.REI.4 Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.

   Derive the quadratic formula from $ax^2 + bx + c = 0$.

   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, an appropriate to the initial form of the equation (limit to real number solutions).

MGSE9-12.A.REI.5 Show and explain why the elimination method works to solve a system of two variable equations.

MGSE9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

MGSE9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

MGSE9-12.A.REI.11 Using graphs, tables, or successive approximations, show that the solution to the equation $f(x) = g(x)$ is the x-value where the y-values of f(x) and g(x) are the same.

MGSE9-12.A.REI.12 Graph the solution set to a linear inequality in two variables.

Interpreting Functions (F.IF)

Understand the concept of a function and use function notation

MGSE9-12.F.IF.1 Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If f is a function, x is the input (an element of the domain), and f(x) is the output (an element of the range). Graphically, the graph is $y = f(x)$.

Understand the concept of a function and use function notation (continued)

MGSE9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MGSE9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (Generally, the scope of high school math defines this subset as the set of natural numbers 1,2,3,4...) By graphing or calculating terms, students should be able to show how the recursive sequence $a_1 = 7$, $a_n = a_{n-1} + 2$; the sequence $s_n = 2(n\text{-}1) + 7$; and the function f(x) = 2x + 5 (when x is a natural number) all define the same sequence.

Interpret functions that arise in applications in terms of the context

MGSE9-12.F.IF.4 Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

MGSE9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

MGSE9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima (as determined by the function or by context).

   b. Graph exponential functions, showing intercepts and end behavior.

MGSE9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and

      interpret these in terms of a context. For example, compare and contrast quadratic functions in standard, vertex, and intercept forms.

MGSE9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.

Building Functions (S.CP)

Build a function that models a relationship between two quantities

MGSE9-12.F.BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression and the recursive process (steps for calculation) from context. For example, if Jimmy starts out with $15 and earns

$2 a day, the explicit expression "2x+15" can be described recursively (either in writing or verbally) as "to find out how much money Jimmy will have tomorrow, you add $2 to his total today." $J_n = J_{n-1} + 2, J_0 = 15$

MGSE9-12.F.BF.2 Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

MGSE9-12.F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and Exponential Models (F.LE)

Construct and compare linear, quadratic, and exponential models and solve problems

MGSE9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

(This can be shown by algebraic proof, with a table showing differences, or by calculating average rates of change over equal intervals).

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Construct and compare linear, quadratic, and exponential models and solve problems continued

MGSE9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MGSE9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model

MGSE9-12.F.LE.5 Interpret the parameters in a linear ($f(x) = mx + b$) and exponential ($f(x) = a \cdot dx$) function in terms of context. (In the functions above, "m" and "b" are the parameters of the linear function, and "a" and "d" are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.

Interpreting Categorical and Quantitative Data (S.ID)

Summarize, represent, and interpret data on a single count or measurement variable

MGSE9-12.S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

MGSE9-12.S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, mean absolute deviation, standard deviation) of two or more different data sets.

MGSE9-12.S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables

MGSE9-12.S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Summarize, represent, and interpret data on two categorical and quantitative variables

MGSE9-12.S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

    a. Decide which type of function is most appropriate by observing graphed data, charted data, or by analysis of context to generate a viable (rough)

       function of best fit. Use this function to solve problems in context. Emphasize linear, quadratic and exponential models.

    b. Using given or collected bivariate data, fit a linear function for a scatter plot that suggests a linear association. Interpret linear models

MGSE9-12.S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

MGSE9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient "r" of a linear fit. (For instance, by looking at a scatterplot, students should be able to tell if the correlation coefficient is positive or negative and give a reasonable estimate of the "r" value.) After calculating the line of best fit

MGSE9-12.S.ID.8 Compute (using technology) and interpret the correlation coefficient "r" of a linear fit. (For instance, by looking at a scatterplot, students should be able to tell if the correlation coefficient is positive or negative and give a reasonable estimate of the "r" value.) After calculating the line of best fit using technology, students should be able to describe how strong the goodness of fit of the regression is, using "r".

MGSE9-12.S.ID.9 Distinguish between correlation and causation.