



Math of Industry and Government (MIG)





Math of Industry & Government (MIG) Teaching & Learning Framework

Semester 1				Semester 2				
Unit 2 6 weeks	Unit 4 3 weeks	Unit 5 5 weeks	Unit 6 4 weeks	Unit 9 4 weeks	Unit 10 3 weeks	Unit 11 3 weeks	Unit 14 4 weeks	Unit 15 4 weeks
Finding Optimal Solutions: Maximization	Finding Optimal Solutions	Integer Programming	Binary Programming	Critical Path Method	Critical Path	Decision Trees	Poisson Distribution	Normal Distribution
N.Q.2 Appropriate Quantities A.SSE.1 Create equations/inequalities A.CED.1-3 Equations/Inequalities through 2-variables with constraints A.REI.12 Graph linear inequalities	N.Q.2 Appropriate Quantities A.SSE.1 Create equations/inequalities A.CED.1-3 Equations/Inequalities through 2-variables with constraints	N.Q.2 Appropriate Quantities A.SSE.1 Create equations/inequalities A.CED.1-3 Equations/Inequalities through 2-variables with constraints	N.Q.2 Appropriate Quantities A.SSE.1 Create equations/inequalities A.CED.1-3 Equations/Inequalities through 2-variables with constraints	N.Q.2 Appropriate Quantities	N.Q.2 Appropriate Quantities	S.CP.2 Independent events S.MD.5-7 Payoff values; expected values; analyze decisions	S.CP.2 Independent events S.MD.2-3 Expected value & probability distribution S.MD.5-7 Payoff values; expected values; analyze decisions	S.ID.1-4 Shape, center & spread; mean & standard deviation S.IC.1-2 Population parameters & simulations S.CP.1-2 Set Theory & independence S.MD.1-7 Expected value; probability distribution; payoff values; fair decisions

These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.
All units will include the Mathematical Practices and indicate skills to maintain

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.
Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities
Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions
Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry
Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions



Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.



Standards for Mathematical Practice (continued)

4. Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.



Standards for Mathematical Practice (continued)

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8. Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics of Industry and Government

This is a course designed to follow the completion of Algebra II, Advanced Algebra, Accelerated Geometry B/Algebra II, or Accelerated Analytic Geometry B/Advanced Algebra. Modeled after operations research courses, Mathematics of Industry and Government allows students to explore decision making in a variety of industries such as: Airline - scheduling planes and crews, pricing tickets, taking reservations, and planning the size of the fleet; Pharmaceutical - R&D management; Logistics companies - routing and planning; Lumber and wood products - managing forests and cutting timber; Local government - deployment of emergency services, and Policy studies and regulation - environmental pollution, air traffic safety, AIDS, and criminal justice policy. Students learn to focus on the development of mathematical models that can be used to model, improve, predict, and optimize real-world systems. These mathematical models include both deterministic models such as mathematical programming, routing or network flows and probabilistic models such as queuing, and simulation. (*Prerequisite: Algebra II, Advanced Algebra, Accelerated Geometry B/Algebra II or Accelerated Analytic Geometry B/Advanced Algebra*)

Instruction and assessment should include the appropriate use of manipulatives and technology. Topics should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic phenomena.



Deterministic Decision Making

Students will explore decision making through linear programming, optimal locations, and optimal paths.

MMIGDD1. Students will use advanced linear programming to make decisions.

- a. Distinguish among continuous, integer, and binary contexts.
- b. Model a contextual problem with three or more variables.
- c. Solve problems with three or more variables using technology. d. Interpret the results.
- e. Examine cause and effect of contextual changes.

MMIGDD2. Students will determine optimal locations and use them to make appropriate decisions.

- a. Find the optimal median location in a one-dimensional context.
- b. Find the optimal median location in a rectilinear context.
- c. Find the optimal location given three equally weighted, non-collinear points. d. Find the optimal location in a set covering context.

MMIGDD3. Students will determine optimal paths and use them to make appropriate decisions.

- a. Relate context to a network representation.
- b. Apply appropriate recursive algorithms for minimum spanning tree, shortest path, and critical path management.
- c. Examine alternate decisions in response to contextual changes.

Probabilistic Decision Making

Students will use normal and other (e.g. binomial, geometric, and Poisson) distributions as well as simulations to make appropriate decisions.

MMIGPD1. Students will use properties of normal distributions to make decisions about optimization and efficiency.

- a. Calculate theoretical and empirical probabilities using standardized and non-standardized data.
- b. Analyze and interpret the probabilities in terms of context.
- c. Consider contextual factors and investigate issues within the decision-making process.
- d. Apply techniques to quality control settings.



Probabilistic Decision Making continued

MMIGPD2. Students will use properties of other distributions (e.g. binomial, geometric, Poisson) to make decisions about optimization and efficiency.

- a. Calculate theoretical and empirical probabilities using standardized and non-standardized data.
- b. Analyze and interpret the probabilities in terms of context.
- c. Consider contextual factors and investigate issues within the decision-making process.

MMIGPD3. Students will use other probabilistic models to make decisions.

- a. Use program evaluation review technique (PERT) to investigate completion times of a project.
- b. Develop and apply transition matrices to make predictions using Markov Chains.
- c. Apply queuing theory.
- d. Consider contextual factors and investigate issues within the decision making process.

MMIGPD4. Students will use computer simulations to make decisions.

- a. Use technology to simulate a real-world situation.
- b. Analyze, evaluate, and interpret results.
- c. Examine alternate decisions in response to contextual changes.

MMIGP5. Students will represent mathematics in multiple ways.

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.