Accelerated Geometry B/Algebra II
Unit 5: Operations with Polynomials

References

Textbook:

- HMH Geometry B/Advanced Algebra, Unit 6: Modules 10 & 11

Online Access:

http://www.my.hrw.com

Helpful Links:

- Khan Academy: https://www.khanacademy.org/math/algebra2/polynomial_and_rational/polynomial_tutorial/v/terms-coefficients-and-exponents-in-a-polynomial
- Binomial Theorem: https://www.khanacademy.org/math/algebra2/polynomial_and_rational/binomial_theorem/v/binomial-theorem
- Dividing Polynomials: https://www.khanacademy.org/math/algebra2/polynomial_and_rational/dividing_polynomials/v/polynomial-division
- Synthetic Division: https://www.khanacademy.org/math/algebra2/polynomial_and_rational/synthetic-division/v/synthetic-division

Dear Parents,

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students will find inverse functions and verify by composition that one function is the inverse of another function.

Concepts Students will Use & Understand

- understand the definition of a polynomial
- interpret the structure and parts of a polynomial expression including terms, factors, and coefficients
- simplify polynomial expressions by performing operations, applying the distributive property, and combining like terms
- use the structure of polynomials to identify ways to rewrite them and write polynomials in equivalent forms to solve problems
- perform arithmetic operations on polynomials and understand how closure applies under addition, subtraction, and multiplication
- divide one polynomial by another using long division
- use Pascal's Triangle to determine coefficients of binomial expansion
- use polynomial identities to solve problems
- use complex numbers in polynomial identities and equations
- find inverses of simple functions

Vocabulary

- **Coefficient:** a number multiplied by a variable.
- **Degree:** the greatest exponent of its variable
- **End Behavior**: the value of f(x) as x approaches positive and negative infinity
- **Pascal's Triangle:** an arrangement of the values of ${}_nC_r$ in a triangular pattern where each row corresponds to a value of n
- **Polynomial:** a mathematical expression involving a sum of nonnegative integer powers in one or more variables multiplied by coefficients. A polynomial in one variable with constant coefficients can be written in $a_nx^n + a_{n-1}x^{n-1} + \ldots a_2x^2 + a_1x + a_0$ form.
- **Remainder Theorem:** states that the remainder of a polynomial f(x) divided by a linear divisor (x – c) is equal to f(c).
- **Synthetic Division:** Synthetic division is a shortcut method for dividing a polynomial by a linear factor of the form (x – a). It can be used in place of the standard long division algorithm.
- **Roots**: solutions to polynomial equations.
- **Zero**: If f(x) is a polynomial function, then the values of x for which f(x) = 0 are called the **zeros** of the function. Graphically, these are the x intercepts.

Sample Problems

Find the following products. Be sure to simplify results.

a. $3x(2x^2+8x+9)$

$6x^3 + 24x^2 + 27x$

b. $-2x^2(5x^2-x-4)$

$-10x^4 + 2x^3 + 8x^2$

c. $(2x+7)(2x-5)$

$4x^2 + 4x - 35$

d. $(4x-7)(3x-2)$

$12x^2 - 29x + 14$

e. $(x-3)(2x^2+3x-1)$

$2x^3 - 3x^2 - 10x + 3$

f. $(6x+4)(x^2-3x+2)$

$6x^3 - 14x^2 + 8$

g. $(4x-7y)(4x+7y)$

$16x^2 - 49y^2$

h. $(3x-4)^2$

$9x^2 - 24x + 16$

i. $(x-1)^3$

$x^3 - 3x^2 + 3x - 1$

j. $(x-1)^4$

$x^4 - 4x^3 + 6x^2 - 4x - 1$

Description	Identity
Difference of Two Squares	$(a+b)(a-b) =$ $a^2 - b^2$
Sum of Two Squares	$(a+bi)(a-bi) =$ $a^2 + b^2$
Perfect Square Trinomial	$(a+b)^2 =$ $a^2 + 2ab + b^2$
Perfect Square Trinomial	$(a-b)^2 =$ $a^2 - 2ab + b^2$
Binomial Cubed	$(a+b)^3 =$ $a^3 + 3a^2b + 3ab^2 + b^3$
Binomial Cubed	$(a-b)^3 =$ $a^3 - 3a^2b + 3ab^2 + b^3$
Sum of Two Cubes	$a^3+b^3 =$ $(a+b)(a^2 - ab + b^2)$
Difference of Two Cubes	$a^3-b^3 =$ $(a-b)(a^2 + ab + b^2)$

Divide: $x-2\overline{)x^3+2x^2-5x-6}$

Long Division vs. Synthetic Division

$$
\begin{array}{r|l}
 & 4x^3 + 5x^2 + 3x + 2 \\
\hline
x-2 & 4x^4 - 3x^3 - 7x^2 - 4x - 9 \\
 & \underline{4x^4 - 8x^3} \\
 & 5x^3 - 7x^2 \\
 & \underline{5x^3 - 10x^2} \\
 & 3x^2 - 4x \\
 & \underline{3x^2 - 6x} \\
 & 2x - 9 \\
 & \underline{2x - 4} \\
 & -5
\end{array}
$$

$$
\begin{array}{r|rrrrr}
 & 4 & 5 & 3 & 2 & \\
\hline
-2 & 4 & -3 & -7 & -4 & -9 \\
 & & \underline{-8} & & & \\
 & & 5 & & & \\
 & & & \underline{-10} & & \\
 & & & 3 & & \\
 & & & & \underline{-6} & \\
 & & & & 2 & \\
 & & & & & \underline{-4} \\
 & & & & & -5
\end{array}
$$