Dear Parents,

In this unit, students continue their study of polynomials by identifying zeros and making connections between zeros of a polynomial and solutions of a polynomial equation. Students will see how the Fundamental Theorem of Algebra can be used to determine the number of solutions of a polynomial equation and will find all the roots of those equations. Students will graph polynomial functions and interpret the key characteristics of the function.

Concepts Students will Use & Understand

- use polynomial identities to solve problems
- use complex numbers in polynomial identities and equations
- understand and apply the rational Root Theorem
- understand and apply the Remainder Theorem
- understand and apply The Fundamental Theorem of Algebra
- understand the relationship between zeros and factors of polynomials
- represent, analyze, and solve polynomial functions algebraically and graphically

Vocabulary

- **End Behavior**: the value of f(x) as x approaches positive and negative infinity
- **Relative Minimum**: a point on the graph where the function is increasing as you move away from the point in the positive and negative direction along the horizontal axis.
- **Relative Maximum**: a point on the graph where the function is decreasing as you move away from the point in the positive and negative direction along the horizontal axis.
- **Fundamental Theorem of Algebra**: every non-zero single-variable polynomial with complex coefficients has exactly as many complex roots as its degree, if each root is counted up to its multiplicity.
- **Multiplicity**: the number of times a root occurs at a given point of a polynomial equation.
- **Pascal’s Triangle**: an arrangement of the values of \( \binom{n}{r} \) in a triangular pattern where each row corresponds to a value of \( n \)
- **Rational Root Theorem**: a theorem that provides a complete list of all possible rational roots of a polynomial equation. It states that every rational zero of the polynomial equation \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 \), where all coefficients are integers, has the following form: \( \frac{P}{q} \), \( P \) is a factor of the constant term \( a_0 \) and \( q \) is a factor of the leading coefficient \( a_n \)
- **Remainder Theorem**: states that the remainder of a polynomial f(x) divided by a linear divisor (x – c) is equal to f(c)
Sample Problems

1. The height of an arrow shot by a 6 foot tall person is given by the function equation where \( h \) is the height and \( t \) is the time. At what time would the arrow be able to hit a target 10 feet in the air?

   The arrow could hit a 10 foot target in 2 sec. or in 2 2/3 sec.

2. Draw a rough sketch of the graph of \( y = -x^2 + 4x - 3 \)

3. A soccer ball is kicked from the ground. The height of the ball is modeled by the equation \( h(t) = -4.9t^2 + 19.6t \)

   Height is in meters. Time is in seconds. How long is the ball in the air?

   4 seconds

4. Describe the key features of the following polynomial function:

   \[ f(x) = x^4 + x^2 - 20 \]

   Rational roots:
   \[ x = -2, 2 \]

   Irrational roots:
   None

   Non-real roots:
   \[ x = -\sqrt{5}i, \sqrt{5}i \]

   Relative maximum points:
   None

   Relative minimum points:
   \( (0, -20) \)

   End behavior:
   \[ x \to -\infty, f(x) \to \infty; \ x \to \infty, f(x) \to \infty \]