Dear Parents,

Students investigate two more types of families of functions – radical and rational functions. They also learn how to simplify expressions and solve equations and inequalities that involve rational and radical expressions. Below you find will several resources that can be used to help support the learning for your child.

Concepts Students will Use & Understand

- Explore Rational and Radical Functions
- Determine rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial
- Notice the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers
- Investigate the properties of simple rational and radical functions and then expand their knowledge of the graphical behavior and characteristics of more complex rational functions
- Recall and make use of their knowledge of polynomial functions as well as compositions of functions to investigate the characteristics of these more complex rational functions
- Solve equations and inequalities involving rational and radical functions
- Understand that not all solutions generated algebraically are actually solutions to the equations and extraneous solutions will be explored
- Apply these rational and radical functions with an emphasis on interpretation of real world phenomena as it relates to certain characteristics of the rational expressions

Vocabulary

- **Radical Function**: A function containing a root. The most common radical functions are the square root and cube root functions: \( f(x) = \sqrt{x} \) and \( g(x) = \sqrt[3]{x} \).
- **Rational Function**: The quotient of two polynomials, \( P(z) \) and \( Q(z) \), where \( R(z) = \frac{P(z)}{Q(z)} \).
Sample Problems

Simplify each expression or solve each equation or inequality.

1a. \( \frac{x + 4}{x^2 - x - 12} + \frac{2x}{x - 4} \)
1b. \( \frac{x + 12}{2x - 5} - \frac{3x - 2}{2x - 5} \)
1c. \( \frac{4x + 16}{2x + 6} - \frac{x^2 + 2x - 3}{x + 4} \)
1d. \( \frac{5x^6}{x^2 y} \div 10x^2 \)
1e. \( \frac{4}{x^2 - 4} = \frac{1}{x - 2} \)
1f. \( \frac{7}{x + 3} < -5 \)

2. \( f(x) = \frac{x^2 + 4x - 5}{x + 1} \)
   a. Zeros:
   ______________________________________________________________________
   b. Vertical asymptote:
   ______________________________________________________________________
   c. Horizontal asymptote:
   ______________________________________________________________________
   d. Graph

Solve each equation.

3a. \( \sqrt{x + 4} = 3\sqrt{x} \)
3b. \( 4(x - 12)^{\frac{1}{3}} = -16 \)
3c. \( \sqrt{-14x + 2} = x - 3 \)

4. Graph: \( f(x) = 3\sqrt{x} + 1 \). State Domain and Range.

5. \( f(x) \) is a radical function whose domain is \( [-\infty, 5] \) and whose range is \( (-\infty, 0] \), and \( g(x) = \sqrt{x + 5} + 2 \). Which statement is true for both \( f(x) \) and \( g(x) \)?
   A. \( g(x) \) has a higher \( y \)-intercept than \( f(x) \)
   B. \( g(x) \) has a greater zero than \( f(x) \)
   C. The functions have the same domain.
   D. The functions have the same range.