



# Algebra II

## Unit 5: Exponential & Logarithmic Functions

### References

**Textbook Connection:**  
HMH Georgia Advanced  
Algebra Text:  
Unit 4: Modules 8 & 9

Every student will receive a text copy and access to the online textbook resource:

<http://my.hrw.com/>

#### Helpful Links:

- GA Virtual:  
<http://cms.gavirtualschool.org/Shared/Math/GSEAdvancedAlgebra/ExponentialAndLogarithms/index.html>
- Regents Prep:  
<http://regentsprep.org/regents/math/algtrig/atp8b/logFunction.htm>
- Khan Academy:  
[https://www.khanacademy.org/math/algebra2/exponential\\_and\\_logarithmic\\_func](https://www.khanacademy.org/math/algebra2/exponential_and_logarithmic_func)
- Purple Math:  
<http://www.purplemath.com/modules/graphlog.htm>
- The Math Page:  
<http://www.themathpage.com/aprecalc/logarithmic-exponential-functions.htm>

### Dear Parents,

Students extend their work with exponential functions to include solving exponential equations with logarithms. They analyze the relationship between these two functions.

In this unit students will:

- Review exponential functions and their graphs
- Explore exponential growth
- Develop the concept of a logarithm as an exponent along with the inverse relationship with exponents
- Define logarithms and natural logarithms
- Develop the change of base formula
- Develop the concept of logarithmic function
- Solving problems relating to exponential functions and logarithms

### Concepts Students will Use & Understand

- The concept of a function
- Various representations of functions
- Exponential functions and characteristics of their graphs
- The solution of linear equations using algebra and graphing approaches
- Familiarity with graphing technology
- Use patterns to write a function to model a situation

### Vocabulary

- **Asymptote:** An asymptote is a line or curve that approaches a given curve arbitrarily closely. A graph never crosses a vertical asymptote, but it may cross a horizontal or oblique asymptote.
- **Common logarithm:** A logarithm with a base of 10. A common logarithm is the exponent,  $a$ , such that  $10^a = b$ . The common logarithm of  $x$  is written  $\log x$ . For example,  $\log 100 = 2$  because  $10^2 = 100$ .
- **Continuously compounded interest:** Interest that is, theoretically, computed and added to the balance of an account each instant. The formula is  $A = Pe^{rt}$ , where  $A$  is the ending amount,  $P$  is the principal or initial amount,  $r$  is the annual interest rate, and  $t$  is the time in years.
- **Compounded interest:** A method of computing the interest, after a specified time, and adding the interest to the balance of the account. Interest can be computed as little as once a year to as many times as one would like. The formula is  $A = P\left(1 + \frac{r}{n}\right)^{nt}$  where  $A$  is the ending amount,  $P$  is the principal or initial amount,  $r$  is the annual interest rate,  $n$  is the number of times compounded per year, and  $t$  is the number of years.
- **Exponential functions:** A function of the form  $y = a^x$  where  $a > 0$  and  $a \neq 1$ .
- **Logarithmic functions:** A function of the form  $y = \log_b x$  with  $b \neq 1$  and  $b$  and  $x$  both positive. A logarithmic function is the inverse of an exponential function. The inverse of  $y = b^x$  is  $y = \log_b x$ .

- **Logarithm:** The logarithm base  $b$  of a number  $x$ ,  $\log_b x$ , is the exponent to which  $b$  must be raised to equal  $x$ .
- **Natural exponential:** Exponential expressions or functions with a base of  $e$ ; i.e.,  $y = e^x$ .
- **Natural logarithm:** A logarithm with a base of  $e$ .  $\ln b$  is the exponent,  $a$ , such that  $e^a = b$ . The natural logarithm of  $x$  is written  $\ln x$  and represents  $\log_e x$ . For example,  $\ln 8 = 2.0794415\dots$  because  $e^{2.0794415\dots} = 8$ .

For further help:

<http://www.teachers.ash.org.au/jeather/maths/dictionary.html>

<http://intermath.coe.uga.edu/dictionary/homepg.asp>

<http://www.amathsdictionaryforkids.com/>

## Sample Practice Problems

1) State the domain and range for  $f(x) = -2^x + 4$

D: all real numbers; R:  $y < 4$

2) State the domain and range for  $3\log_5 x$

D:  $x > 0$ ; R: all real numbers

3) Solve  $2(3)^{2x} = 5$

$x = 0.417$

4) Solve  $5\log(x - 2) = 11$

$x = 160.49$

5) Identify asymptotes, y-intercept and point of maximum growth:  $y = \frac{2}{1 + e^{-2x}}$

Asymptotes: x-axis and  $y=2$ ; y-intercept  $(0,1)$ ; maximum growth is at  $(0,1)$

6) Find the inverse of the function  $y = \log_6 x$

$y = 6^x$

7) The value of a new car purchased for \$24,900 decreases by 10% per year. Write an exponential decay model for the value of the car. After about how many years will the car be worth half its purchase price?

$V(t) = 24,900(0.90)^t$ ; about 6.58 years

8) You deposit \$4,000 in an account that pays 7% annual interest compounded continuously. Find the balance at the end of five years.

\$5,676.27