



# College Readiness Mathematics







## Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

### **1. Make sense of problems and persevere in solving them.**

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **2. Reason abstractly and quantitatively.**

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

### **3. Construct viable arguments and critique the reasoning of others.**

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.



## Mathematics | Standards for Mathematical Practice (continued)

### 4. Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### 5. Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

### 6. Attend to precision.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

**7. Look for and make use of structure.** By high school, students look closely to discern a pattern or structure. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

### 8. Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.



## Quantities (N.Q.)

### Reason quantitatively and use units to solve problems.

**MGSE9-12.N.Q.** Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems:

- Identify, use, and record appropriate units of measure within context, within data displays, and on graphs;
- Convert units and rates using dimensional analysis (English-to-English and Metric-to-Metric without conversion factor provided and between English and Metric with conversion factor);
- Use units within multi-step problems and formulas; interpret units of input and resulting units of output.

**MGSE9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.

## Seeing Structure in Expression (A.SSE)

### Interpret the structure of expressions.

**MGSE9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MGSE9-12.A.SSE.2** Use the structure of an expression to rewrite it in different equivalent forms.

For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .

### Write expressions in equivalent forms to solve problems.

**MGSE9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

## Creating Equations (A.CED)

### Create equations that describe numbers or relationships.

**MGSE9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions (integer inputs only).



**MGSE9-12.A.CED.2** Create linear, quadratic, and exponential equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (The phrase “in two or more variables” refers to formulas like the compound interest formula, in which  $A = P(1 + r/n)^nt$  has multiple variables.)

**MGSE9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints.

**MGSE9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations.

*Examples: Rearrange Ohm’s law  $V = IR$  to highlight resistance  $R$ ; Rearrange area of a circle formula  $A = \pi r^2$  to highlight the radius  $r$ .*

## Reasoning with Equations and Inequalities (A.REI)

### Understand solving equations as a process of reasoning and explain the reasoning.

**MGSE9-12.A.REI.1** Using algebraic properties and the properties of real numbers, justify the steps of a simple, one-solution equation. Students should justify their own steps, or if given two or more steps of an equation, explain the progression from one step to the next using properties.

**MGSE9-12.A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

### Solve equations and inequalities in one variable.

**MGSE9-12.A.REI.3** Solve linear equations and inequalities in one variable including equations with coefficients represented by letters.

*For example, given  $ax + 3 = 7$ , solve for  $x$ .*

**MGSE9-12.A.REI.4** Solve quadratic equations in one variable.

### Solve systems of equations.

**MGSE9-12.A.REI.5** Show and explain why the elimination method works to solve a system of two-variable equations.

**MGSE9-12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.



**MGSE9-12.A.REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*

### **Represent and solve equations and inequalities graphically.**

**MGSE9-12.A.REI.11** Using graphs, tables, or successive approximations, show that the solution to the equation  $f(x) = g(x)$  is the  $x$ -value where the  $y$ -values of  $f(x)$  and  $g(x)$  are the same.

**MGSE9-12.A.REI.12** Graph the solution set to a linear inequality in two variables.

## **Interpreting Functions (F.IF)**

### **Understand the concept of a function and use function notation.**

**MGSE9-12.F.IF.1** Understand that a function from one set (the input, called the domain) to another set (the output, called the range) assigns to each element of the domain exactly one element of the range, i.e. each input value maps to exactly one output value. If  $f$  is a function,  $x$  is the input (an element of the domain), and  $f(x)$  is the output (an element of the range). Graphically, the graph is  $y = f(x)$ .

### **Interpret functions that arise in applications in terms of the context.**

**MGSE9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

**MGSE9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.*

**MGSE9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.



## **Analyze functions using different representations.**

**MGSE9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology.

**MGSE9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

**MGSE9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.*

## **Building Functions (F.BF)**

### **Build a function that models a relationship between two quantities.**

**MGSE9-12.F.BF.1** Write a function that describes a relationship between two quantities.

**MGSE9-12.F.BF.2** Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

### **Build new functions from existing functions.**

**MGSE9-12.F.BF.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

## **Linear, Quadratic, and Exponential Models (F.LE)**

### **Construct and compare linear, quadratic, and exponential models and solve problems.**

**MGSE9-12.F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.

**MGSE9-12.F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).



**MGSE9-12.F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

### **Interpret expressions for functions in terms of the situation they model.**

**MGSE9-12.F.LE.5** Interpret the parameters in a linear ( $f(x) = mx + b$ ) and exponential ( $f(x) = a \cdot dx$ ) function in terms of context. (In the functions above, “m” and “b” are the parameters of the linear function, and “a” and “d” are the parameters of the exponential function.) In context, students should describe what these parameters mean in terms of change and starting value.

### **Use coordinates to prove simple geometric theorems algebraically.**

**MGSE9-12.G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0,2)$ .* (Focus on quadrilaterals, right triangles, and circles.)

**MGSE9-12.G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

## **Geometric Measurement and Dimension (G.GMD)**

### **Explain volume formulas and use them to solve problems.**

**MGSE9-12.G.GMD.1** Give informal arguments for geometric formulas.

- a. Give informal arguments for the formulas of the circumference of a circle and area of a circle using dissection arguments and informal limit arguments.
- b. Give informal arguments for the formula of the volume of a cylinder, pyramid, and cone using Cavalieri’s principle.

**MGSE9-12.G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.



## Modeling with Geometry (G.MG)

### Apply geometric concepts in modeling situations.

**MGSE9-12.G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

**MGSE9-12.G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

## Interpreting Categorical and Quantitative Data (S.ID)

### Summarize, represent, and interpret data on a single count or measurement variable.

**MGSE9-12.S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).

**MGSE9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, mean absolute deviation, standard deviation) of two or more different data sets.

**MGSE9-12.S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

### Summarize, represent, and interpret data on two categorical and quantitative variables.

**MGSE9-12.S.ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

**MGSE9-12.S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

### Interpret linear models.

**MGSE9-12.S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.



**MGSE9-12.S.ID.8** Compute (using technology) and interpret the correlation coefficient “ $r$ ” of a linear fit. (For instance, by looking at a scatterplot, students should be able to tell if the correlation coefficient is positive or negative and give a reasonable estimate of the “ $r$ ” value.) After calculating the line of best fit using technology, students should be able to describe how strong the goodness of fit of the regression is, using “ $r$ ”.

**MGSE9-12.S.ID.9** Distinguish between correlation and causation.

## **Making Inferences and Justifying Conclusions (S.IC)**

### **Understand and evaluate random processes underlying statistical experiments.**

**MGSE9-12.S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

### **Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**

**MGSE9-12.S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.