

Clarification of Standards for Parents
Grade 3 Mathematics Unit 3

Dear Parents,

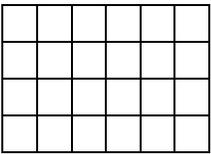
We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Three. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions.
☺

MGSE3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

This standard references various strategies that can be used to solve word problems involving multiplication and division. Students should apply their skills to solve word problems. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many cookies does each person receive? ($4 \times 9 = 36$, $36 \div 6 = 6$).

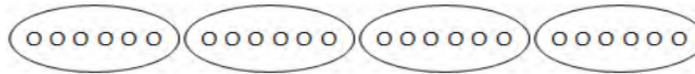
Examples of multiplication: There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there?

This task can be solved by drawing an array by putting 6 desks in each row. This is an array model:



This task can also be solved by drawing pictures of equal groups.

4 groups of 6 equals 24 objects

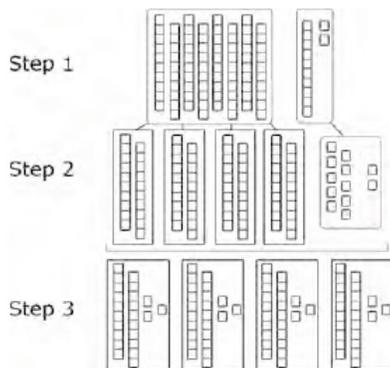


A student could also reason through the problem mentally or verbally, "I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom." A number line could also be used to show equal jumps. Third grade students should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables). Letters are also introduced to represent unknowns in third grade.

Examples of division: There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class. ($\square \div 4 = 6$. *There are 24 students in the class*).

Determining the number of objects in each share (partitive division, where the size of the groups is unknown):

Example: The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



Determining the number of shares (measurement division, where the number of groups is unknown):

Example: Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	$24 - 4 =$ 20	$20 - 4 =$ 16	$16 - 4 =$ 12	$12 - 4 =$ 8	$8 - 4 =$ 4	$4 - 4 =$ 0

Solution: The bananas will last for 6 days.

MGSE3.OA.5 Apply properties of operations as strategies to multiply and divide.

Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.)

$3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.)

Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

Students focused on the commutative property of multiplication and the associative property of multiplication in unit two. In unit three, the focus shifts to the distributive property.

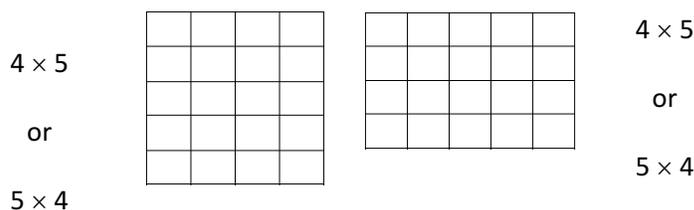
This standard references properties (rules about how numbers work) of multiplication. While students DO NOT need to use the formal terms of these properties, students should understand that properties are rules about how numbers work, they need to be flexibly and fluently applying each of them. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.

The **associative property** states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies $7 \times 5 \times 2$, a student could rearrange the numbers to first multiply $5 \times 2 = 10$ and then multiply $10 \times 7 = 70$.

The **commutative property** (order property) states that the order of numbers does not matter when you are adding or multiplying numbers. For example, if a student knows that $5 \times 4 = 20$, then they also know that $4 \times 5 = 20$. The array below could be described as a 5×4 array for 5 columns and 4 rows, or a 4×5 array for 4 rows and 5 columns. There is no “fixed” way to write the dimensions of an array as rows \times columns or columns \times rows.

Students should have flexibility in being able to describe both dimensions of an array.

Example:



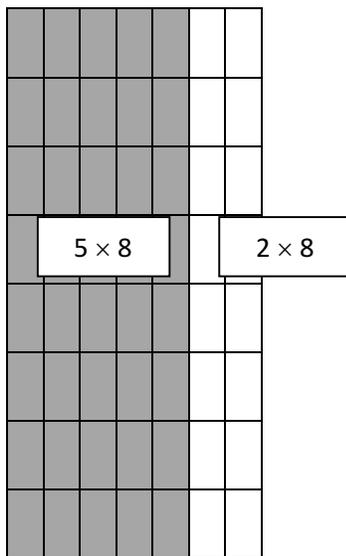
Students are introduced to the **distributive property of multiplication over addition** as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product. Here are ways that students could use the distributive property to determine the product of 7×6 . Again, students should use the distributive property, but can refer to this in informal language such as “breaking numbers apart”.

Student 1
7×6
$7 \times 5 = 35$
$7 \times 1 = 7$
$35 + 7 = 42$

Student 2
7×6
$7 \times 3 = 21$
$7 \times 3 = 21$
$21 + 21 = 42$

Student 3
7×6
$5 \times 6 = 30$
$2 \times 6 = 12$
$30 + 12 = 42$

Another example if the distributive property helps students determine the products and factors of problems by breaking numbers apart. For example, for the problem $7 \times 8 = ?$, students can decompose the 7 into a 5 and 2, and reach the answer by multiplying $5 \times 8 = 40$ and $2 \times 8 = 16$ and adding the two products ($40 + 16 = 56$).



To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- $0 \times 7 = 7 \times 0 = 0$ (Zero Property of Multiplication)
- $1 \times 9 = 9 \times 1 = 9$ (Multiplicative Identity Property of 1)
- $3 \times 6 = 6 \times 3$ (Commutative Property)
- $8 \div 2 = 2 \div 8$ (Students are only to determine that these are not equal)
- $2 \times 3 \times 5 = 6 \times 5$
- $10 \times 2 < 5 \times 2 \times 2$
- $2 \times 3 \times 5 = 10 \times 3$
- $0 \times 6 > 3 \times 0 \times 2$

MGSE3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). “Know from memory” should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9×9).

By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategies students may use to attain fluency include:

- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
- Tens facts (relating to place value, 5×10 is 5 tens or 50)
- Five facts (half of tens)
- Skip counting (counting groups of ___ and knowing how many groups have been counted)
- Square numbers (ex: 3×3)
- Nines (10 groups less one group, e.g., 9×3 is 10 groups of 3 minus one group of 3)
- Decomposing into known facts (6×7 is 6×6 plus one more group of 6)
- Turn-around facts (Commutative Property)
- Fact families (Ex: $6 \times 4 = 24$; $24 \div 6 = 4$; $24 \div 4 = 6$; $4 \times 6 = 24$)
- Missing factors

General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

MGSE3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

This standard refers to two-step word problems using the four operations. The size of the numbers should be limited. Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100.

This standard calls for students to represent problems using equations with a letter to represent unknown quantities.

Example: Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution ($2 \times 5 + m = 25$).

This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.

Example: Here are some typical estimation strategies for the problem:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

Student 1

I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.

Student 2

I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.

Student 3

I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

MGSE3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

This standard also mentions identifying patterns related to the properties of operations.

Examples:

- Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends ($14 = 7 + 7$).
- Multiples of even numbers (2, 4, 6, and 8) are always even numbers.
- On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
- On an addition chart, the sums in each row and column increase by the same amount.

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What do you notice about the numbers highlighted in pink in the multiplication table? Explain a pattern using properties of operations. *When one changes the order of the factors (commutative property), they will still get the same product; example $6 \times 5 = 30$ and $5 \times 6 = 30$.*

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?

Student: The product will always be an even number.

Teacher: Why?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way?

Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.

Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	19	11	12	13	14	15	16	17	18	19	20

- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
□	□	□
□	□	□
□	□	□
20	0	20

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

MGSE3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

This standard extends students’ work in multiplication by having them apply their understanding of place value.

This standard expects that students go beyond tricks that hinder understanding such as “just adding zeros” and explain and reason about their products. For example, for the problem 50×4 , students should think of this as 4 groups of 5 tens or 20 tens. Twenty tens equals 200.

MGSE3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. This standard continues throughout the third grade year.

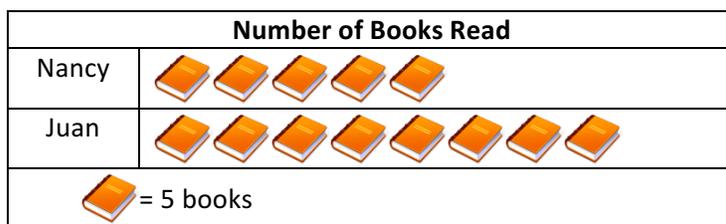
Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should Pose a question, Collect data, Analyze data, and Interpret data (PCAI). Students should be graphing data that is relevant to their lives

Example:

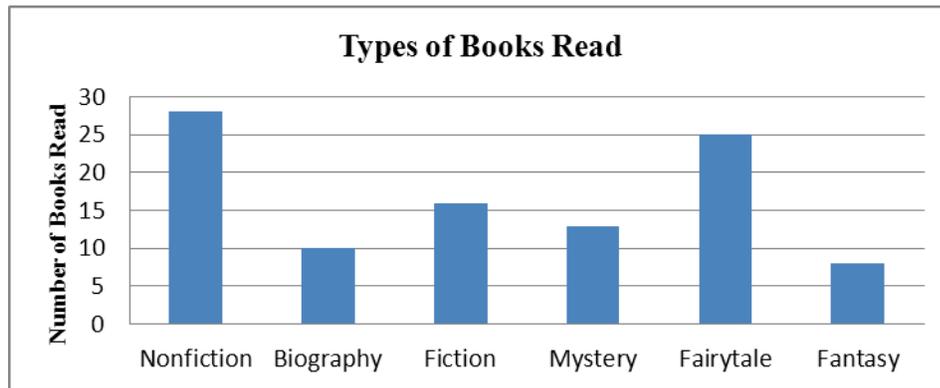
Pose a question: Student should come up with a question. What is the typical genre read in our class?

Collect and organize data: student survey

Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.



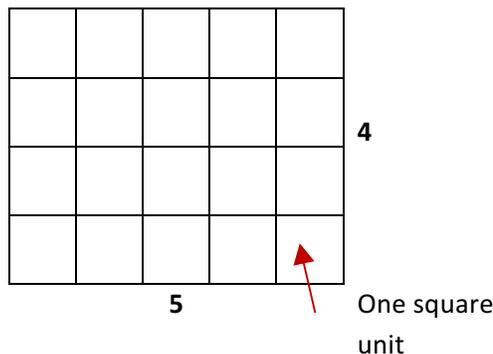
Analyze and Interpret data:

- How many more nonfiction books were read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library which would be the best genre? Why?

MGSE3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

- A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
- A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

These standards call for students to explore the concept of covering a region with “unit squares,” which could include square tiles or shading on grid or graph paper.



MGSE3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.

MGSE3.MD.7 Relate area to the operations of multiplication and addition.

- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Students should tile rectangles then multiply their side lengths to show it is the same.

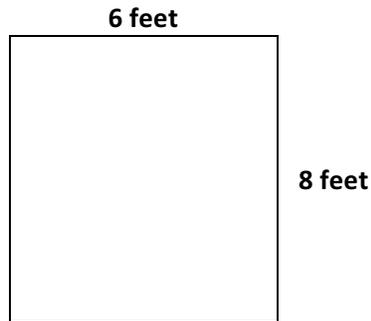
To find the area, one could count the squares or multiply $3 \times 4 = 12$.

1	2	3	4
5	6	7	8
9	10	11	12

- b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.**

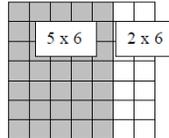
Students should solve real world and mathematical problems

Example: Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?

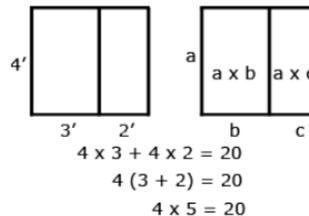


- c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.**

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7×6 figure can be determined by finding the area of a 5×6 and 2×6 and adding the two sums.



Example:



Common Misconceptions

Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

(Adapted from Henry County Schools)