Clarification of Standards for Parents
Grade 4 Mathematics Unit 2

Dear Parents,

We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Two. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

MGSE4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.

Examples:

5 x 8 = 40: Sally is five years old. Her mom is eight times older. How old is Sally's Mom?

5 x 5 = 25: Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

This standard calls for students to translate comparative situations into equations with an unknown and solve.

Examples:

- **Unknown Product:** A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? *(3 × 6 = p)*
- **Group Size Unknown:** A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? *(18 ÷ p = 3 or 3 × p = 18)*
- **Number of Groups Unknown:** A red scarf costs \$18. A blue scarf costs \$6. How many times as much does the red scarf cost compared to the blue scarf? *(18 ÷ 6 = p or 6 × p = 18)*

  When distinguishing multiplicative comparison from additive comparison, students should note the following.
- Additive comparisons focus on the difference between two quantities.
  - For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?
  - A simple way to remember this is, "How many more?"
- Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other.
  - For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?

  A simple way to remember this is "How many times as much?" or "How many times as many?"

MGSE4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Some typical estimation strategies for this problem are shown below.

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	**Student 2**
First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\ ^{2}/_{6}$

Possible solutions:

- Problem **A: 7.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

- Problem **B: 7 r 2.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

- Problem **C: 8.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

- Problem **D: 7 or 8.**
  Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.*

- Problem **E:** $\mathbf{7\,{}^{2}/_{6}}$**.**
  Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $\mathbf{7}\,{}^{2}/_{6}$
  Example:
  There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)
- **Clustering around an average** (When the values are close together an average value is selected and multiplied by the number of values to determine an estimate.)
- **Rounding and adjusting** (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)
- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)
- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

Common Misconceptions

A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.

Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.

Prime vs. Composite:

- A prime number is a number greater than 1 that has only 2 factors, 1 and itself.
- Composite numbers have more than 2 factors.
- Students investigate whether numbers are prime or composite by building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g., 7 can be made into only 2 rectangles, 1 × 7 and 7 × 1, therefore it is a prime number).
- Finding factors of the number. Students should understand the process of finding factor pairs so they can do this for any number 1-100.

Example:
Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.
Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:
Factors of 24: 1, 2, 3, 4, 6,8, 12, 24
Multiples: 1, 2, 3, 4, 5, … , <u>24</u>
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, <u>24</u>
3, 6, 9, 12, 15, 15, 21, <u>24</u>
4, 8, 12, 16, 20, <u>24</u>
8, 16, <u>24</u>
12, <u>24</u>
<u>24</u>

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- All even numbers are multiples of 2.
- All even numbers that can be halved twice (with a whole number result) are multiples of 4.
- All numbers ending in 0 or 5 are multiples of 5.

<u>**MGSE4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.** ***For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.***</u>

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23,28, .	Start with 3; add 5	The numbers alternately end with a 3 or an 8
5, 10, 15, 20, …	Start with 5; add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that 3nd with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:
Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 8, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 – 1 = 2, 9 – 3 = 6, 27 – 9 = 18, etc.).

This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

Day	Operation	Beans
0	$3 \times 0 + 4$	4
1	$3 \times 1 + 4$	7
2	$3 \times 2 + 4$	10
3	$3 \times 3 + 4$	13
4	$3 \times 4 + 4$	16
5	$3 \times 5 + 4$	19

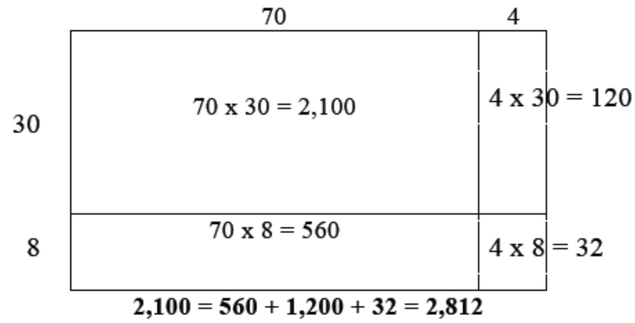
MGSE4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. **Use of the standard algorithm for multiplication is an expectation in the fifth grade.**

This standard calls for students to multiply numbers using a variety of strategies.

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

Student 1	Student 2	Student 3
25×12 I broke 12 up into 10 and 2. $25 \times 10 = 250$ $25 \times 2 = 50$ $250 + 50 = 300$	25×12 I broke 25 into 5 groups of 5. $5 \times 12 = 60$ I have 5 groups of 5 in 25. $60 \times 5 = 300$	25×12 I doubled 25 and cut 12 in half to get 50×6. $50 \times 6 = 300$

Example:
What would an array area model of 74 x 38 look like?

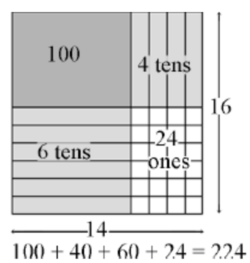


Examples:
To illustrate 154 × 6, students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property,

154 × 6 = (100 + 50 + 4) × 6
= (100 × 6) + (50 × 6) + (4 × 6)
=600 + 300 + 24 = 924.

The area model below shows the partial products for 14 x 16 = 224.
Using the area model, students first verbalize their understanding:

- 10 × 10 is 100
- 4 × 10 is 40
- 10 × 6 is 60, and
- 4 × 6 is 24.



Students use different strategies to record this type of thinking.

Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

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  25
× 24
 400 (20 × 20)
 100 (20 × 5)
 80 (4 × 20)
 20 (4 × 5)
600
```

MGSE.4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Example:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- **Using Base 10 Blocks:** Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- **Using Place Value:** 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)
- **Using Multiplication:** 4 × 50 = 200, 4 × 10 = 40, 4 × 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65

This standard calls for students to explore division through various strategies.

Example:

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

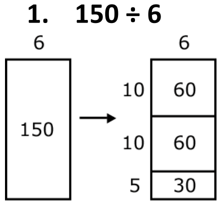
Student 1	**Student 2**	**Student 3**
592 divided by 8 There are 70 eights in 560. 592 – 560 = 32 There are 4 eights in 32. 70 + 4 = 74	592 divided by 8 I know that 10 eights is 80. If I take out 50 eights that is 400. 592 – 400 = 192 I can take out 20 more eights which is 160. 192 – 160 = 32 8 goes into 32 four times. I have none left. I took out 50, then 20 more, then 4 more. That's 74.	I want to get to 592. 8 × 25 = 200 8 × 25 = 200 8 × 25 = 200 200 + 200 + 200 = 600 600 – 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams.

Example:

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

1. 150 ÷ 6



Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, "6 times what number is a number close to 150?" They recognize that 6 × 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left, they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 × 5 is 30, they write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Student express their calculations in various ways:

   a. 150
   <u>–60</u> (6 × 10)
   90
   <u>–60</u> (6 × 10)
   30
   <u>–30</u> (6 × 5)
   0

   150 ÷ 6 = 10 + 10 + 5 = 25

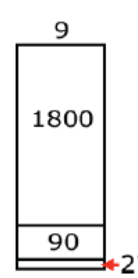
   b. 150 ÷ 6 = (60 ÷ 6) + (60 ÷ 6) + (30 ÷ 6) = 10 + 10 + 5 = 25

Example:

1917 ÷ 9

A student's description of his or her thinking may be:

I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines. 1917 ÷ 9 = 213.



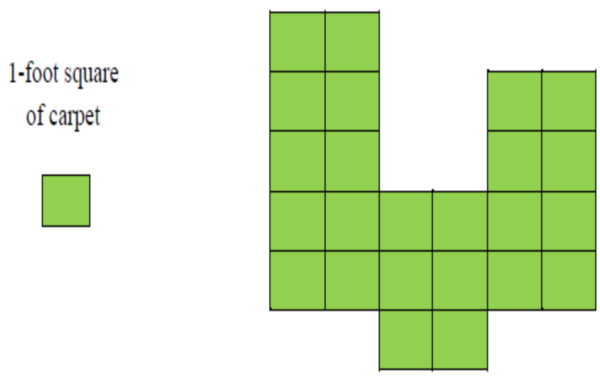
Common Misconceptions
Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

<u>MGSE4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.</u> ***<u>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</u>***

Students developed understanding of area and perimeter in 3rd grade by using visual models.

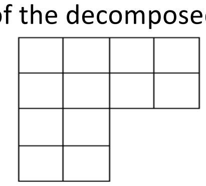
While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is *l* x *w* and the answer will always be in square units. The formula for perimeter can be 2 *l* + 2 *w* or 2 (*l* + *w*) and the answer will be in linear units. This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization.

Example:
Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?

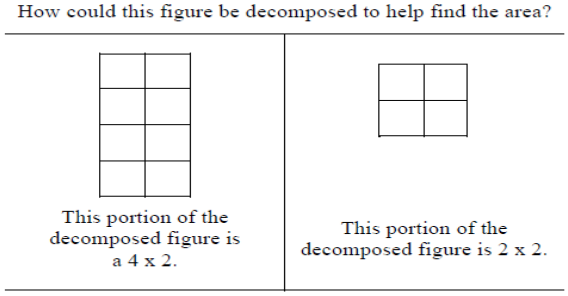


<u>MGSE4.MD.8 Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</u>

This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles. Students can decompose a rectilinear figure into different rectangles. They find the area of the rectilinear figure by adding the areas of each of the decomposed rectangles together.



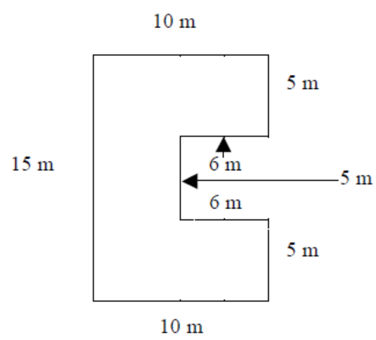
How could this figure be decomposed to help find the area?



4 x 2 = 8 and 2 x 2 = 4
So 8 + 4 = 12
Therefore the total area of this figure is 12 square units

Example:

A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?



I can divide this figure into three smaller rectangles.

First: 10 m x 5 m = 50 m^2

Second: 5 m x 4 m = 20 m^2

Third: 10 m x 5 m = 50 m^2

50 m^2 + 50 m^2 + 20 m^2 = 120 m^2

(Adapted from Henry County Schools)