<u>Clarification of Standards for Parents</u>
<u>Grade 5 Mathematics Unit 1</u>

Dear Parents,
We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit One. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

<u>MGSE5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</u>
The standard calls for students to evaluate expressions with parentheses ( ), brackets [ ] or braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However at this level students are to only simplify the expressions because there are no variables.

Bill McCallum, state standards author, states: *In general students in Grade 5 will be using parentheses only, because the convention about nesting that you describe is quite common, and it's quite possible that instructional materials at this level wouldn't even mention brackets and braces. However, the nesting order is only a convention, not a mathematical law. It's important to distinguish between mathematical laws (e.g. the commutative law) and conventions of notation (e.g. nesting of parentheses). Some conventions of notation are important enough that you want to insist on them in the classroom (e.g. order of operations). But I don't think correct nesting of parentheses falls into that category. The main point of the standard is to understand the structure of numerical expressions with grouping symbols.*

In other words- evaluate expressions with brackets **or** braces **or** parentheses. No nesting at 5^{th} grade.
This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions.
Examples:

- (26 + 18) 4 — Solution: 11
- 12 – (0.4 × 2) — Solution: 11.2
- (2 + 3) × (1.5 – 0.5) — Solution: 5
- 6– (1/2 + 1/3) — Solution: 516

To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently.
Example:

- 15 – 7 – 2 = 10 → 15 – (7 – 2) = 10
- Compare 3 × 2 + 5 and 3 × (2 + 5).
- Compare 15 – 6 + 7 and 15 – (6 + 7).

<u>MGSE5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them</u>
This standard refers to expressions. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equals sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1).

Example:

- 4(5 + 3) is an expression.
- When we compute 4(5 + 3) we are evaluating the expression. The expression equals 32.
- 4(5 + 3) = 32 is an equation.

This standard calls for students to verbally describe the relationship between expressions without actually calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations.

Example:
Write: Write an expression for the steps "double five and then add 26.
Student: *(2 × 5) + 26*

Interpret: Describe how the expression 5(10 × 10) relates to 10 × 10.
Student: The expression 5(10 × 10) is 5 times larger than the expression 10 × 10 since I know that I that 5(10 × 10) means that I have 5 groups of (10 × 10).

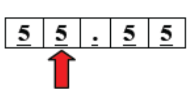
<u>Common Misconceptions</u>
Students may believe the order in which a problem with mixed operations is written is the order to solve the problem. Allow students to use calculators to determine the value of the expression, and then discuss the order the calculator used to evaluate the expression. Do this with four-function and scientific calculators.

<u>MGSE5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</u>
This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is $1/10^{th}$ the size of the tens place. In 4^{th} grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons. Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left.

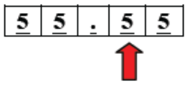
Example:
The 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542 the value of the 2 is 10 times greater. Meanwhile, the 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542 the value of the 4 in the number 324 is $1/10^{th}$ of its value in the number 542.

Example:
A student thinks, "I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is $1/10^{th}$ of the value of a 5 in the hundreds place.
Based on the base-10 number system, digits to the left are times as great as digits to the right; likewise, digits to the right are $1/10^{th}$ of digits to the left. For example, the 8 in 845 has a value of 800 which is ten times as much as the 8 in the number 782. In the same spirit, the 8 in 782 is $1/10^{th}$ the value of the 8 in 845.
To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe $1/10^{th}$ of that model using fractional language. ("This is 1 out of 10 equal parts. So it is 1/10. I can write this using 1/10 or 0.1.") They repeat the process by finding 1/10 of a 1/10 (e.g., dividing 1/10 into 10 equal parts to arrive at 1/100 or 0.01) and can explain their reasoning: "0.01 is 1/10 of 1/10 thus is 1/100 of the whole unit."

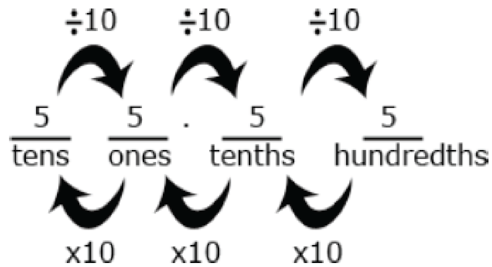
In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.



The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times five tenths.



The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.



<u>MGSE5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</u>

This standard includes multiplying by multiples of 10 and powers of 10, including 10^2 which is 10×10=100, and 10^3 which is $10 \times 10 \times 10$ =1,000. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10. It is a shift in the digits.

Examples:

$2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$

Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point shifts to the right.

$350 \div 103 = 350 \div 1{,}000 = 0.350 = 0.35$

$^{350}/_{10} = 35$ $(350 \times {}^{1}/_{10})$ $^{35}/_{10} = 3.5$

$(35 \times {}^{1}/_{10})$ $^{3.5}/_{10} = 0.35$ $(3.5 \times {}^{1}/_{10})$

This will relate well to subsequent work with operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is shifting (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point will shift to the left.

Students need to be provided with opportunities to explore this concept and come to this understanding; this should not just be taught procedurally.

Examples:

Students might write:

- $36 \times 10 = 36 \times 10^1 = 360$
- $36 \times 10 \times 10 = 36 \times 10^2 = 3600$
- $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36{,}000$
- $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360{,}000$

Students might think and/or say:
I noticed that every time, I multiplied by 10 I placed a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to shift it one place value to the left.

When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to place a zero at the end to have the 3 tens represent 3 one-hundreds (instead of 3 tens) and the 6 ones represents 6 tens (instead of 6 ones).

Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense.
$523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places.
$5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places.
$52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place.

MGSE5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm (or other strategies demonstrating understanding of multiplication) up to a 3 digit by 2 digit factor.

This standard refers to fluency which means accuracy (correct answer), efficiency (a reasonable amount of steps), and flexibility (using strategies such as the distributive property or breaking numbers apart also using strategies according to the numbers in the problem, 26 × 4 may lend itself to (25 × 4) + 4 where as another problem might lend itself to making an equivalent problem 32 × 4 = 64 × 2. This standard builds upon students' work with multiplying numbers in 3rd and 4th grade. In 4th grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding. The size of the numbers should NOT exceed a three-digit factor by a two-digit factor.

Examples of alternative strategies:
There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1	**Student 2**	**Student 3**
225 × 12 I broke 12 up into 10 and 2. 225 × 10 = 2,250 225 × 2 = 450 2,250 + 450 = 2,700	225 × 12 I broke 225 up into 200 and 25. 200 × 12 = 2,400 I broke 25 up into 5 × 5, so I had 5 × 5 × 12 or 5 × 12 × 5. 5 × 12 = 60 60 × 5 = 300 Then I added 2,400 and 300. 2,400 + 300 = 2,700	I doubled 225 and cut 12 in half to get 450 × 6. Then I doubled 450 again and cut 6 in half to 900 × 3. 900 × 3 = 2,700

Example:
Draw an array model for 225 × 12 → 200 × 10, 200 × 2, 20 × 10, 20 × 2, 5 × 10, 5 × 2.

225 x 12

	200	20	5
10	2,000	200	50
2	400	40	10

```
  2,000
    400
    200
     40
     50
+    10
  2,700
```

MGSE5.NBT.6 Fluently divide up to 4-digit dividends and 2-digit divisors by using at least one of the following methods: strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations or concrete models. (e.g., rectangular arrays, area models)

This standard references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is critical. Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In 4th grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:

There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?

Student 1

1,716 ÷ 16

There are 100 16's in 1,716.

1,716 – 1,600 = 116

I know there are at least 6 16's in 116.

116 – 96 = 20

I can take out one more 16.

20 – 16 = 4

There were 107 teams with 4 students left over. If we put the extra students on different teams, 4 teams will have 17 students.

Student 2

1,716 ÷ 16

There are 100 16's in 1,716.

Ten groups of 16 is 160. That's too big. Half of that is 80, which is 5 groups.

I know that 2 groups of 16's is 32.

I have 4 students left over.

1,716 – 1,600	100
116 – 80	5
36 – 32	2
4	

Student 3

1,716 ÷ 16

I want to get to 1,716. I know that 100 16's equals 1,600. I know that 5 16's equals 80.

1,600 + 80 = 1,680

Two more groups of 16's equals 32, which gets us to 1,712. I am 4 away from 1,716.

So we had 100 + 6 + 1 = 107 teams. Those other 4 students can just hang out.

Student 4

How many 16's are in 1,716?

We have an area of 1,716. I know that one side of my array is 16 units long. I used 16 as the height. I am trying to answer the question: What is the width of my rectangle if the area is 1,716 and the height is 16?

	100	7
16	100 × 16 = 1,600	7 × 16 = 112

1,716 – 1,600 = 116

116 – 112 = 4

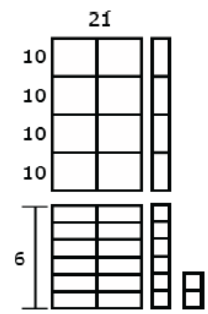
100 + 7 = 107 R 4

Examples:

- Using expanded notation: 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25
- Using understanding of the relationship between 100 and 25, a student might think:
    - I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
    - 600 divided by 25 has to be 24.
    - Since 3 × 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5. (Note that a student might divide into 82 and not 80.)
    - I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
    - 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.
- Using an equation that relates division to multiplication, 25 × *n* = 2682, a student might estimate the answer to be slightly larger than 100 because s/he recognizes that 25 × 100 = 2500.

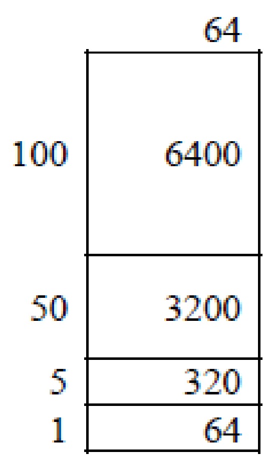
Example: 968 ÷ 21

Using base ten models, a student can represent 962 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.



Example: 9984 ÷ 64

An area model for division is shown below. As the student uses the area model, he or she keeps track of how much of the 9984 is left to divide.



$$
\begin{array}{rl}
64\overline{)9984} & \\
-6400 & (100 \times 64) \\
\hline
3584 & \\
-3200 & (50 \times 64) \\
\hline
384 & \\
-320 & (5 \times 64) \\
\hline
64 & \\
-64 & (1 \times 64) \\
\hline
0 &
\end{array}
$$

(Adapted from Henry County Schools)