COBB TEACHING AND LEARNING STANDARDS

Statistical Reasoning

Cobb County School District

ONE TEAM ONE GOAL STUDENT SUCCESS

Statistical Reasoning Teaching & Learning Framework

Semester 1						Semester 2				
Unit 1 3 weeks	**Unit 2** 3 weeks	**Unit 3** 3 weeks	**Unit 4** 3 weeks	**Unit 5** 3 weeks	**Unit 6** 3 weeks	**Unit 7** 3 weeks	**Unit 8** 3 weeks	**Unit 9** 3 weeks	**Unit 10** 3 weeks	**Unit 11** 3 weeks
Statistical Problem Solving Process	**Formulating Questions**	**Collecting Data**	**The Role of Random-ness**	**Analyzing Data**	**Comparing Distributions**	**Bivariate Comparisons**	**Interpreting Results & Inference**	**Simulations & Margin of Error**	**Simulations & P-Value**	**Creating Experiments & Culminating Project**
MSRFQ1 Students will apply the statistical method to real-world situations; MSRCD3. Students will distinguish between the three types of study designs for collecting data (sample survey, experiment, and observational study) and will know the scope of the interpretation for each design type.	**MSRFQ2.** Students will identify whether the data are categorical or quantitative (numerical).	**MSRCD1.** Students will distinguish between a population distribution, a sample data distribution, and a sampling distribution.	**MSRCD2.** Students will understand that randomness should be incorporated into a sampling or experimental procedure. **MSRCD4.** Students will distinguish between the role of randomness and the role of sample size with respect to using a statistic from a sample to estimate a population parameter.	**MSRAD1**. Students will use distributions to identify the key features of the data collected.	**MSRAD2.** Students will use distributions to compare two or more groups.	**MSRAD3.** Students will determine if an association exists between two variables (pattern or trend in bivariate data) and use values of one variable to predict values of another variable.	**MSRIR1.** Students will ask if the difference between two sample proportions or two sample means is due to random variation or if the difference is significant.	**MSRIR2.** Students will understand that when randomness is incorporated into a sampling or experimental procedure, probability provides a way to describe the 'long-run' behavior of a statistic as described by its sampling distribution.	**MSRIR2.** Students will understand that when randomness is incorporated into a sampling or experimental procedure, probability provides a way to describe the 'long-run' behavior of a statistic as described by its sampling distribution.	All standards for the course

These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.
All units will include the Mathematical Practices and indicate skills to maintain

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematics | Standards for Mathematical Practice (continued)

4. Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures

8. Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Course Description: Statistical Reasoning

This course teaches students how to use four-steps of the statistical process in the context of sports: ask questions, collect data, analyze data, and make conclusions. Major statistical topics include: analyzing distributions of univariate and bivariate data, both categorical and numerical, using graphs and summary statistics; correlation and least squares regression; using simulations to estimate probability distributions; theoretical probability distributions, including the binomial and normal distributions; rules of probability, including conditional probability and expected value; the logic of hypothesis testing, including stating hypotheses, calculating and interpreting *p*-values, drawing conclusions, and Type I and Type II errors; using confidence intervals to estimate parameters; and proper methods of data collection, including sampling and experimentation. Use of technology, including online applets and the graphing calculator will be prominent in the course. Throughout the course, students will complete investigations that require students to complete the four-step statistical process using athletes of their choice.

Pre-Requisite

Passing grade in Advanced Algebra

Context for Course

In Georgia, students are required to take four years of mathematics. Additionally, many majors at most colleges expect students to take a course in statistical reasoning. This 21^{st} century skill is a reflection of the increasingly data driven world that we live in. The purpose of this course, *Statistical Reasoning*, is to provide students with a class that introduces them to statistical reasoning in a context that is rich with hands on examples designed to spark student interest.

Formulate Questions

Students will formulate questions to clarify the problem at hand and formulate one (or more) questions that can be answered with data.

MSRFQ1. Students will apply the statistical method to real-world situations.

a. Formulate questions to clarify the problem at hand and formulate one (or more) questions that can be answered with data.
b. Collect data by designing a plan to collect appropriate data and employ the plan to collect the data.
c. Analyze data by selecting appropriate graphical and numerical methods and using these methods to analyze the data.
d. Interpret results by interpreting the analysis and relating the interpretation to the original question.

MSRFQ2. Students will identify whether the data are categorical or quantitative (numerical).

Students will be able to identify the difference between categorical and quantitative (numerical) data.

a. Determine the appropriate graphical display for each type of data.
b. Determine the type of data used to produce a given graphical display.

Collect Data

<u>Students will design and implement a plan to collect the appropriate data to answer the statistical question.</u>

MSRCD1. Students will distinguish between a population distribution, a sample data distribution, and a sampling distribution.

- a. Students will identify the three types of distributions.
  - i. Recognize a population distribution has fixed values of its parameters that are usually unknown.
  - ii. Recognize a sample data distribution is taken from a population distribution and the data distribution is what is seen in practice hoping it approximates the population distribution.
  - iii. Recognize a sampling distribution is the distribution of a sample statistic (such as a sample mean or a sample proportion) obtained from repeated samples. The sampling distribution provides the key for determining how close to expect a sample statistic approximates the population parameter.
- b. Students will create sample data distributions and a sampling distribution.
  - i. Create a sample data distribution by taking a sample from a defined population and summarizing the data in a distribution.
  - ii. Create a sampling distribution of a statistic by taking repeated samples from a population (either hands-on or by simulation with technology).

MSRCD2. Students will understand that randomness should be incorporated into a sampling or experimental procedure.
Students will be able to implement a reasonable random method for selecting a sample or for assigning treatments in an experiment.

- a. Implement a simple random sample.
- b. Randomly assign treatments to experimental subjects or objects.

MSRCD3. Students will distinguish between the three types of study designs for collecting data (sample survey, experiment, and observational study) and will know the scope of the interpretation for each design type.
Students will be able to distinguish between the three types of study designs for collecting data (sample survey, experiment, and observational study) and know the scope of the interpretation for each design type.

- a. Determine the type of study design appropriate for answering a statistical question.
- b. Determine the appropriate scope of inference for the study design used.

MSRCD4. Students will distinguish between the role of randomness and the role of sample size with respect to using a statistic from a sample to estimate population parameter**.**
Students will be able to distinguish the roles of randomization and sample size with designing studies.

- a. Recognize that randomization reduces bias where bias occurs when certain outcomes are systematically more likely to appear.
- b. Recognize that random selection from a population plays a different role than random assignment in an experiment.
- c. Recognize that sample size impacts the precision with which estimates of the population parameters can be made (larger the sample size the more precision).

Analyze Data

Students will select appropriate graphical and numerical methods and use these methods to analyze the data.

MSRAD1. Students will use distributions to identify the key features of the data collected.
Students will describe the distribution for quantitative and categorical data.

- a. Describe the distribution for quantitative data.
  - i. Describe and interpret the shape of the distribution.
  - ii. Describe and interpret the measures of center for the distribution.
  - iii. Describe and interpret the patterns in variability for the distribution.
  - iv. Describe and interpret any outliers or gaps in the distribution.
- b. Describe the distribution for categorical data.
  - i. Describe and interpret the modal category for the distribution.
  - ii. Describe and interpret patterns that exist for the distribution.

MSRAD2. Students will use distributions to compare two or more groups.
Students will compare two or more groups by analyzing distributions.

- a. Construct appropriate graphical displays of distributions.
- b. Use graphical and numerical attributes of distributions to make comparisons between distributions.

MSRAD3. Students will determine if an association exists between two variables (pattern or trend in bivariate data) and use values of one variable to predict values of another variable.
Students will analyze associations between variables and make predictions from one variable to another.

- a. Analyze associations between two variables.
  - i. Create scatter plots for two-variable numerical data.
  - ii. Create two-way tables for two-variable categorical data.
  - iii. Analyze patterns and trends in data displays.
- b. Make predictions and draw conclusions from two-variable data based on data displays.
- c. Distinguish between association and causation.

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Interpret Results

Students will interpret results and make connections to the original question.

MSRIR1. Students will ask if the difference between two sample proportions or two sample means is due to random variation or if the difference is significant.
Students will be able to determine if there are differences between two population parameters or treatment effects.
a. Using simulation, determine the appropriate model to decide if there is a difference between two population parameters.
b. Using simulation, determine the appropriate model to decide if there is a difference between two treatment effects.

MSRIR2. Students will understand that when randomness is incorporated into a sampling or experimental procedure, probability provides a way to describe the 'long-run' behavior of a statistic as described by its sampling distribution.
Students will be able to create simulated sampling distributions and understand how to use the sampling distribution to make predictions about a population parameter(s) or the difference in treatment effects.
a. Create an appropriate simulated sampling distribution (using technology) and develop a margin of error.
b. Create an appropriate simulated sampling distribution (using technology) and develop a p-value.