



# Accelerated Pre-Calculus

## Unit 7: Vectors

### References

**Textbook Connection:**  
**Foerster PreCalculus Text:**  
**Chapters 11-12**

### Helpful Links:

GA Virtual:

<http://cms.gavirtualschool.org/Shared/Math/GSEPreCalculus/Vectors/index.html>

Teach Mathematics:

<http://www.teachmathematics.net/activities/dancing-vectors.htm>

Better Explained:

<http://betterexplained.com/articles/vector-calculus-understanding-the-dot-product/>

<http://betterexplained.com/articles/measure-any-distance-with-the-pythagorean-theorem/>

Faraday:

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Add2Vectors.html>

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Add3Vectors.html>

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Subtract2Vectors.html>

### Dear Parents,

Students will extend their understanding of complex numbers and their operations through graphical representations. Students will perform operations on vectors and use the operations to represent various quantities.

### Concepts Students will Use & Understand

- Perform arithmetic operations with complex numbers.
- Represent complex numbers & their operations on the complex plane.
- Represent & model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices & use matrices in applications.

### Vocabulary

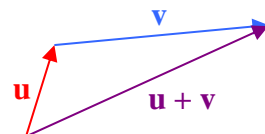
- **Vector:** A mathematical object that has both magnitude and direction. Vectors can be expressed as  $\mathbf{v}$ , or  $\langle a, b \rangle$ , or as a directed line segment (arrow) in the plane.
- **Scalar:** A real number. A scalar has magnitude but not direction.
- **Initial Point:** The point at the “tail” of the arrow representing a vector. *Often, the initial point is assumed to be  $(0, 0)$ . This is the case in the notation  $\langle a, b \rangle$ .*
- **Terminal Point:** The point at the “tip” of the arrow representing a vector.
- **Magnitude of a Vector:** The distance between a vector’s initial and terminal points,

denoted  $||\mathbf{v}||$  or  $|\mathbf{v}|$ .  $||\mathbf{v}|| = ||\langle a, b \rangle|| = \sqrt{a^2 + b^2}$

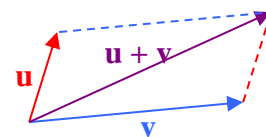
*Also called the length, norm, or absolute value of the vector.*

- **Components of a Vector:**  $a$  and  $b$  in the vector  $\langle a, b \rangle$ .
- **Parallel Vectors:** Two or more vectors whose directions are the same or opposite.
- **Equivalent Vectors:** Two or more vectors that have the same direction and magnitude—i.e., whose representations are the same in the form  $\langle a, b \rangle$ . *Note that equivalent vectors may not have the same initial and terminal points.*
- **Zero Vector:** The vector  $\langle 0, 0 \rangle$ .
- **Resultant Vector:** The vector that results from adding two or more vectors.

**Tail-to-Head Representation:** A geometric representation of vector addition  $\mathbf{u} + \mathbf{v}$  wherein the initial point of  $\mathbf{v}$  is placed at the terminal point of  $\mathbf{u}$ . The vector beginning at the initial point of  $\mathbf{u}$  and ending at the (translated) terminal point of  $\mathbf{v}$  represents  $\mathbf{u} + \mathbf{v}$ .



- **Parallelogram Representation / Parallelogram Rule:** A geometric representation of vector addition  $\mathbf{u} + \mathbf{v}$  wherein a parallelogram is formed by placing the initial points of  $\mathbf{u}$  and  $\mathbf{v}$  at the same place and letting each vector represent the sides of



a parallelogram. The diagonal of the resulting parallelogram, starting at this shared initial point, represents  $\mathbf{u} + \mathbf{v}$ .

- **Velocity:** A vector whose magnitude is an object's speed (a scalar) and whose direction is the direction of the object's motion.

*Note that speed is a scalar—magnitude, no direction—whereas velocity tells us how fast an object is moving and in what direction.*

- **Complex Plane:** A 2-dimensional representation of complex numbers established by a horizontal real axis and a vertical imaginary axis.
- **Rectangular Form of a Complex Number:**  $a + bi$
- **cis  $\vartheta$ :** Shorthand for  $\cos \vartheta + i \sin \vartheta$
- **Polar Form of a Complex Number:**  $r (\cos \vartheta + i \sin \vartheta) = r \text{cis } \vartheta$
- **Complex Conjugate of  $z = a + bi$ :**  $\bar{z} = a - bi$
- **Modulus of a Complex Number:** The distance between a number and 0 when plotted

on the complex plane:  $|z| = |a + bi| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$

*Also called absolute value or magnitude.*

- **Argument of  $z$ ,  $\arg(z)$ :** The angle—typically chosen in  $(-\pi, \pi]$ —formed by the positive-real axis and a segment connecting  $z$  to 0 in the complex plane.
- **Re( $z$ ):**  $a$ , the real part of the complex number  $z = a + bi$   
**Im( $z$ ):**  $b$ , the coefficient of the imaginary part of the complex number  $z = a + bi$

## Sample Practice Problems

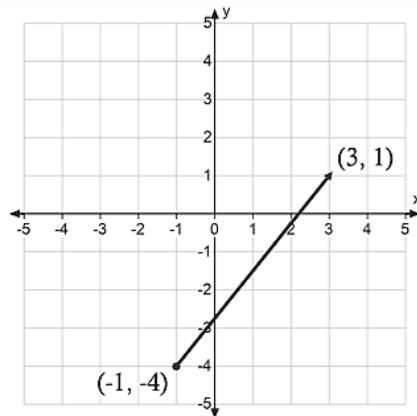
1. Given:  $|\vec{u}| = 14$ ,  $|\vec{v}| = 9$ , &  $\vec{u} \parallel \vec{v}$ , find  $|\vec{u} + \vec{v}|$ .

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2. A ship leaving port sails for 100 miles in a direction  $40^\circ$  north of due east. Find the magnitude of the vertical and horizontal components.

About 64 miles, about 77 miles

3. Find the component form of the given vector.



$\langle 4, 5 \rangle$

4. What is the cross product of  $\mathbf{a} = \langle 1, 2, 3 \rangle$  and  $\mathbf{b} = \langle 4, 5, 6 \rangle$ ?

$\mathbf{a} \times \mathbf{b} = \langle -3, 6, -3 \rangle$

5. Find the magnitude & direction of the sum of  $\langle 4, 5 \rangle$  and  $\langle -1, 4 \rangle$ . Magnitude:  $3\sqrt{10}$ ;

Direction:  $71.57^\circ$