Dear Parents,

In this unit students will:

- represent and manipulate data using matrices
- define the order of a matrix as the number of rows by the number of columns
- add and subtract matrices and know these operations are possible only when the dimensions are equal
- recognize that matrix addition and subtraction are commutative
- multiply matrices by a scalar and understand the distributive and associative properties apply to matrices
- multiply matrices and know when the operation is defined
- recognize that matrix multiplication is not commutative
- understand and apply the properties of a zero matrix
- understand and apply the properties of an identity matrix
- find the determinant of a square matrix and understand that it is a nonzero value if and only if the matrix has an inverse
- use 2 X 2 matrices as transformations of a plane and determine the area of the plane using the determinant
- write a system of linear equations as a matrix equation and use the inverse of the coefficient matrix to solve the system
- write and use vertex-edge graphs to solve problems

Concepts Students will Use & Understand

- Commutative Property
- Associative Property
- Distributive Property
- Identity Properties of Addition and Multiplication
- Inverse Properties of Addition and Multiplication
- Solving Systems of Equations Graphically and Algebraically

Vocabulary

- **Determinant**: the product of the elements on the main diagonal minus the product of the elements off the main diagonal
- **Dimensions or Order of a Matrix**: the number of rows by the number of columns
- **Identity Matrix**: the matrix that has 1’s on the main diagonal and 0’s elsewhere
- **Inverse Matrices**: matrices whose product (in both orders) is the Identity matrix
- **Matrix**: a rectangular arrangement of numbers into rows and columns
- **Scalar**: in matrix algebra, a real number is called a scalar
- **Square Matrix**: a matrix with the same number of rows and columns
- **Zero Matrix**: a matrix whose entries are all zeros

For further help:

- [http://intermath.coe.uga.edu/dictnary/homepg.asp](http://intermath.coe.uga.edu/dictnary/homepg.asp)
**Properties**

Let \( a, b, \) and \( c \) be real numbers

<table>
<thead>
<tr>
<th>Addition Properties</th>
<th>Multiplication Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a + b = b + a )</td>
<td>( ab = ba )</td>
</tr>
<tr>
<td>( (a + b) + c = a + (b + c) )</td>
<td>( (ab)c = a(bc) )</td>
</tr>
</tbody>
</table>

**Commutative**

**Associative**

**Identity**

There exists a unique real number zero, 0, such that \( a + 0 = 0 + a = a \)

**Inverse**

For each real number \( a \), there is a unique real number \( \frac{1}{a} \) such that \( a + (-a) = (-a) + a = 0 \)

**Sample Problems**

1. Find the dimensions:

\[
\begin{bmatrix}
-3 & 5 \\
4 & 1/4 \\
-\pi & 0
\end{bmatrix}
\]

3 rows, 2 columns; Dimensions: 3 x 2

2. Two stores carry small, medium, and large sweatshirts. The table shows the inventory at the stores. Arrange the data in a matrix. Give the dimensions of the matrix.

<table>
<thead>
<tr>
<th>Sweatshirt Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Small</td>
</tr>
<tr>
<td>Medium</td>
</tr>
<tr>
<td>Large</td>
</tr>
<tr>
<td>Store A</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>21</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>Store B</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>32</td>
</tr>
<tr>
<td>28</td>
</tr>
</tbody>
</table>

The dimensions are 2 x 3

3. Multiply the following matrix:

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{bmatrix} \times \begin{bmatrix}
7 & 8 \\
9 & 10 \\
11 & 12
\end{bmatrix}
\]

\[
\begin{bmatrix}
58 & 64 \\
139 & 154
\end{bmatrix}
\]

4. Find the inverse of the following matrix:

\[
\begin{bmatrix}
1 & 0 & 1 \\
1 & 1 & 9 \\
0 & 1 & 9
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 1 & -1 \\
-1 & 9 & -8 \\
1 & -1 & 1
\end{bmatrix}
\]

The inverse exists!

5. What system of equations is represented by the matrix equation?

\[
\begin{bmatrix}
-41 & 1 & 0 \\
1 & 50 & 1 \\
67 & 4 & 0
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
1 \\
7 \\
-29
\end{bmatrix}
\]

\[
-41x + y = 1 \\
x + 50y + z = 7 \\
67x + 4y = -29
\]