



# Pre-Calculus

## Unit 5: Trigonometric Identities

### References

**Textbook Connection:**  
Glencoe PreCalculus Text:  
Chapter 5

Every student will receive a text copy and access to the online textbook resource:  
<http://my.hrw.com/>

#### Helpful Links:

- [Addition Identities](#)
- [Khan Academy Addition & Double Angle for Sine/Cosine](#)
- [Double & Half Angles](#)
- [All Identities](#)
- [All Identities Practice](#)

### Dear Parents,

In this unit students will:

- build upon their work with trigonometric identities with addition and subtraction formulas
- will look at addition and subtraction formulas geometrically
- prove addition and subtraction formulas
- use addition and subtraction formulas to solve problems

### Concepts Students will Use & Understand

- Demonstrate a method to prove addition or subtraction identities for sine, cosine, and tangent.
- Apply addition or subtraction identities for sine, cosine, and tangent.
- Use addition or subtraction identities to find missing values for sine, cosine and tangent functions.

### Vocabulary

- **Addition Identity for Cosine:**  $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- **Addition Identity for Sine:**  $\sin(x + y) = \sin x \cos y + \cos x \sin y$
- **Addition Identity for Tangent:**  $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- **Double Angle Identity for Sine:**  $\sin(2x) = 2 \sin x \cos x$
- **Double Angle Identity for Cosine:**  
 $\cos(2x) = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$
- **Double Angle Identity for Tangent:**  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
- **Half Angle Identity for Sine:**  $\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$
- **Half Angle Identity for Cosine:**  $\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$
- **Half Angle Identity for Tangent:**  $\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$
- **Identity:** an identity is a relation that is always true, no matter the value of the variable.
- **Subtraction Identity for Cosine:**  $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- **Subtraction Identity for Sine:**  $\sin(x - y) = \sin x \cos y - \cos x \sin y$
- **Subtraction Identity for Tangent:**  $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

For further help:

- <http://www.teachers.ash.org.au/jeather/maths/dictionary.html>
- <http://intermath.coe.uga.edu/dictionary/homepg.asp>
- <http://www.amathsdictionaryforkids.com/>

## Sample Practice Problems

- Find the exact value of each trigonometric expression.
  - $\cos 75^\circ$
  - $\sin (-210^\circ)$
  - $\tan \frac{\pi}{12}$
- Find the exact value of each expression.
  - $\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ$
  - $\frac{\tan 48^\circ + \tan 12^\circ}{1 - \tan 48^\circ \tan 12^\circ}$
- Simplify each expression.
  - $\sin 3y \cos y + \cos 3y \sin y$
  - $\frac{\tan 5\theta + \tan \theta}{\tan 5\theta \tan \theta - 1}$
- Find the values of  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$  for the given value and interval.
 
$$\tan \theta = \sqrt{3}, \left(0, \frac{\pi}{2}\right)$$
- Solve each equation on the interval  $[0, 2\pi]$ .
  - $\sin 2\theta = \cos \theta$
  - $\tan 2\theta - \tan 2\theta \tan^2 \theta = 2$
- Find the exact value of each expression.
  - $\sin 67.5^\circ$
  - $\cos \frac{\pi}{12}$
- Solve each equations on the interval  $[0, 2\pi]$ .
  - $\sin \frac{\theta}{2} + \cos \theta = 1$
  - $\tan \frac{\theta}{2} = \sin \frac{\theta}{2}$

### Answer Key

- |  |                                     |                    |
|--|-------------------------------------|--------------------|
| 1a. $\frac{\sqrt{6}-\sqrt{2}}{4}$                                  | 1b. $\frac{1}{2}$                   | 1c. $2 - \sqrt{3}$ |
| 2a. 1  | 2b. $\sqrt{3}$                      |                    |
| 3a. $\sin 4y$  | 3b. $-\tan 6\theta$                 |                    |
| 4. $\frac{\sqrt{3}}{2}; -\frac{1}{2}; -\sqrt{3}$                   |                                     |                    |
| 5a. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ | 5b. $\frac{\pi}{4}, \frac{5\pi}{4}$ |                    |
| 6a. $\frac{\sqrt{2+\sqrt{2}}}{2}$                                  | 6b. $\frac{\sqrt{2+\sqrt{3}}}{2}$   |                    |
| 7a. $0, \frac{\pi}{3}, \frac{5\pi}{3}$                             | 7b. 0                               |                    |