



Pre-Calculus

Unit 7: Vectors

References

Textbook Connection:
Glencoe PreCalculus Text:
Chapter 8

Every student will receive a text copy and access to the online textbook resource:
<http://my.hrw.com/>

Helpful Links:

GA Virtual:

<http://cms.gavirtualschool.org/Shared/Math/GSEPrecalculus/Vectors/index.html>

Teach Mathematics:

<http://www.teachmathematics.net/activities/dancing-vectors.htm>

Better Explained:

<http://betterexplained.com/articles/vector-calculus-understanding-the-dot-product/>

<http://betterexplained.com/articles/measure-any-distance-with-the-pythagorean-theorem/>

Faraday:

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Add2Vectors.html>

<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Add3Vectors.html>
<http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Vectors/Subtract2Vectors.html>

Dear Parents,

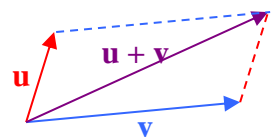
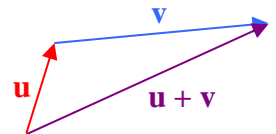
Students will extend their understanding of complex numbers and their operations through graphical representations. Students will perform operations on vectors and use the operations to represent various quantities.

Concepts Students will Use & Understand

- Perform arithmetic operations with complex numbers.
- Represent complex numbers & their operations on the complex plane.
- Represent & model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices & use matrices in applications.

Vocabulary

- **Vector:** A mathematical object that has both magnitude and direction. Vectors can be expressed as \mathbf{v} , or $\langle a, b \rangle$, or as a directed line segment (arrow) in the plane.
- **Scalar:** A real number. A scalar has magnitude but not direction.
- **Initial Point:** The point at the “tail” of the arrow representing a vector. *Often, the initial point is assumed to be $(0, 0)$. This is the case in the notation $\langle a, b \rangle$.*
- **Terminal Point:** The point at the “tip” of the arrow representing a vector.
- **Magnitude of a Vector:** The distance between a vector’s initial and terminal points, denoted $||\mathbf{v}||$ or $|\mathbf{v}|$. $||\mathbf{v}|| = ||\langle a, b \rangle|| = \sqrt{a^2 + b^2}$
Also called the length, norm, or absolute value of the vector.
- **Components of a Vector:** a and b in the vector $\langle a, b \rangle$.
- **Parallel Vectors:** Two or more vectors whose directions are the same or opposite.
- **Equivalent Vectors:** Two or more vectors that have the same direction and magnitude—i.e., whose representations are the same in the form $\langle a, b \rangle$. *Note that equivalent vectors may not have the same initial and terminal points.*
- **Zero Vector:** The vector $\langle 0, 0 \rangle$.
- **Resultant Vector:** The vector that results from adding two or more vectors.
Tail-to-Head Representation: A geometric representation of vector addition $\mathbf{u} + \mathbf{v}$ wherein the initial point of \mathbf{v} is placed at the terminal point of \mathbf{u} . The vector beginning at the initial point of \mathbf{u} and ending at the (translated) terminal point of \mathbf{v} represents $\mathbf{u} + \mathbf{v}$.
- **Parallelogram Representation / Parallelogram Rule:** A geometric representation of vector addition $\mathbf{u} + \mathbf{v}$ wherein a parallelogram is formed by placing the initial points of \mathbf{u} and \mathbf{v} at the same place and letting each vector represent the sides of



a parallelogram. The diagonal of the resulting parallelogram, starting at this shared initial point, represents $\mathbf{u} + \mathbf{v}$.

- **Velocity:** A vector whose magnitude is an object's speed (a scalar) and whose direction is the direction of the object's motion.

Note that speed is a scalar—magnitude, no direction—whereas velocity tells us how fast an object is moving and in what direction.

- **Complex Plane:** A 2-dimensional representation of complex numbers established by a horizontal real axis and a vertical imaginary axis.
- **Rectangular Form of a Complex Number:** $a + bi$
- **cis ϑ :** Shorthand for $\cos \vartheta + i \sin \vartheta$
- **Polar Form of a Complex Number:** $r (\cos \vartheta + i \sin \vartheta) = r \text{ cis } \vartheta$
- **Complex Conjugate of $z = a + bi$:** $\bar{z} = a - bi$
- **Modulus of a Complex Number:** The distance between a number and 0 when plotted

on the complex plane: $|z| = |a + bi| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$

Also called absolute value or magnitude.

- **Argument of z , $\arg(z)$:** The angle—typically chosen in $(-\pi, \pi]$ —formed by the positive-real axis and a segment connecting z to 0 in the complex plane.
- **Re(z):** a , the real part of the complex number $z = a + bi$
Im(z): b , the coefficient of the imaginary part of the complex number $z = a + bi$

Sample Practice Problems

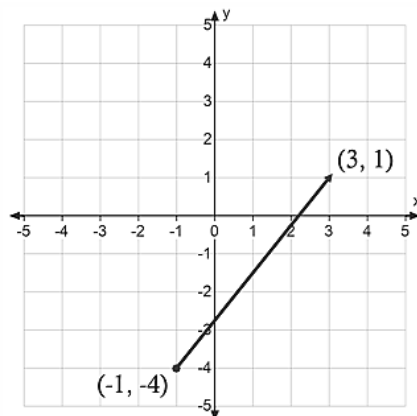
1. Given: $|\vec{u}| = 14$, $|\vec{v}| = 9$, & $\vec{u} \parallel \vec{v}$, find $|\vec{u} + \vec{v}|$.

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2. A ship leaving port sails for 100 miles in a direction 40° north of due east. Find the magnitude of the vertical and horizontal components.

About 64 miles, about 77 miles

3. Find the component form of the given vector.



$\langle 4, 5 \rangle$

4. What is the cross product of $\mathbf{a} = \langle 1, 2, 3 \rangle$ and $\mathbf{b} = \langle 4, 5, 6 \rangle$?

$\mathbf{a} \times \mathbf{b} = \langle -3, 6, -3 \rangle$

5. Find the magnitude & direction of the sum of $\langle 4, 5 \rangle$ and $\langle -1, 4 \rangle$. Magnitude: $3\sqrt{10}$;

Direction: 71.57°