Dear Parents,

In this unit students will:

- Synthesize and generalize what they have learned about a variety of function families
- Derive the formula for the sum of a finite geometric series and use it to solve problems
- Explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions
- Identify appropriate types of functions to model a situation,
- Adjust parameters to improve the model,
- Compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit
- Determine whether it is best to model with multiple functions creating a piecewise function

Concepts Students will Use & Understand

- Quantitative reasoning
- Solving various functions (finding zeros) through factoring, using other algebraic processes, using geometry, or by graphing
- Properties of exponents and the associated properties of logarithms
- A working knowledge of geometric vocabulary
- Writing explicit and recursive formulas for geometric sequences
- The ability to recall and apply basic algebraic and geometric processes
- An ability to understand mathematics through a variety of representations
- Familiarity with technology, particularly the graphing calculator
- Prior knowledge and understanding of functions learned earlier in the course, as this is the culminating unit

Vocabulary

- **Geometric Sequence**: is a sequence with a constant ratio between successive terms
- **Geometric Series**: the expression formed by adding the terms of a geometric sequence
- **Recursive**: A type of sequence in which successive terms are generated by preceding terms in the sequence.
- **Sum of a finite geometric series**: The sum, \( S_n \), of the first \( n \) terms of a geometric sequence is given by \( S_n = \frac{a_1 - a_1 r^n}{1 - r} = \frac{a_1 (1 - r^n)}{1 - r} \), where \( a_1 \) is the first term and \( r \) is the common ratio (\( r \neq 1 \)).
- **Sum of an infinite geometric series**: The general formula for the sum \( S \) of an infinite geometric series \( a_1 + a_2 + a_3 + \ldots \) with common ratio \( r \) where \( |r| < 1 \) is \( S = \frac{a_1}{1 - r} \). If an
infinite geometric series has a sum, i.e. if $|r| < 1$, then the series is called a **convergent** geometric series. All other geometric (and arithmetic) series are **divergent**.

For further help:

**Sample Practice Problems**

1. The price of dance lesson depends upon the number of lessons that you select. If $x$ is the number of lessons then the fee for the lessons (in dollars) can be found using the piecewise function
   \[ f(x) = \begin{cases} 
   40x & \text{if } 0 < x \leq 4 \\
   30x & \text{if } 4 < x \leq 8 \\
   25x & \text{if } x > 8 
   \end{cases} \]
   The lessons are increasing by 10% per lesson with a $5 processing fee for each student. What is the new function for the cost of lessons?
   \[ f(x) = \begin{cases} 
   44x + 5 & \text{if } 0 < x \leq 4 \\
   33x + 5 & \text{if } 4 < x \leq 8 \\
   27.5x + 5 & \text{if } x > 8 
   \end{cases} \]

2. Use the function $f(x) = \frac{\sqrt{5}x}{3}$ to answer the following questions:
   - **Domain**: all real numbers
   - **Range**: all real numbers
   - What is the inverse of $f(x)$?
     \[ f^{-1}(x) = \frac{x^3}{5} \]
   - **Domain**: all real numbers
   - **Range**: all real numbers
   - Over what line does the function and its inverse reflect across on the coordinate plane?
     \[ y = x \]

3. Identify the axis of symmetry, vertex, intercepts, domain, range, slope, and max/min of the following absolute value function:
   \[ f(x) = |x| + 3 \]
   - **A.O.S.**: $x = 0$
   - **Vertex**: $(0, 3)$
   - **x-intercept**: none
   - **y-intercept**: 3
   - **Domain**: all real numbers
   - **Range**: $y \geq 3$
   - **Left Slope**: -1
   - **Right Slope**: 1
   - **Minimum**: 3