Algebra II
Unit 6: Mathematical Modeling

References

Textbook Connection:
HMH Advanced Algebra Text:
Unit 2: Module 5 & Unit 6:
Modules 12-15

Every student will receive a text copy and access to the online textbook resource:
http://my.hrw.com/

Helpful Links:

- ☐ GA Virtual: http://cms.gavirtualschool.org/Shared/Math/GSEAdvancedAlgebra/MathematicalModeling/index.html
- ☐ Khan Academy: https://www.khanacademy.org/math/algebra2/functions-and-graphs/piecewise-functions-tutorial/v/graphs-of-absolute-value-functions
- ☐ Math Bits Notebook: https://mathbitsnotebook.com/Algebra2/Sequences/SSGeometric.html
- ☐ Math Bits Notebook: http://mathbitsnotebook.com/Algebra1/FunctionGraphs/FNGTypePiecewise.html
- ☐ Wolfram Math World: http://mathworld.wolfram.com/GeometricS

Dear Parents,

In this unit students will:

- Synthesize and generalize what they have learned about a variety of function families
- derive the formula for the sum of a finite geometric series and use it to solve problems
- Explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions
- Identify appropriate types of functions to model a situation,
- Adjust parameters to improve the model,
- Compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit
- Determine whether it is best to model with multiple functions creating a piecewise function

Concepts Students will Use & Understand

- quantitative reasoning
- solving various functions (finding zeros) through factoring, using other algebraic processes, using geometry, or by graphing
- properties of exponents and the associated properties of logarithms
- a working knowledge of geometric vocabulary
- writing explicit and recursive formulas for geometric sequences
- the ability to recall and apply basic algebraic and geometric processes
- an ability to understand mathematics through a variety of representations
- familiarity with technology, particularly the graphing calculator
- prior knowledge and understanding of functions learned earlier in the course, as this is the culminating unit

Vocabulary

- **Geometric Sequence:** is a sequence with a constant ratio between successive terms
- **Geometric Series:** the expression formed by adding the terms of a geometric sequence
- **Recursive**: A type of sequence in which successive terms are generated by preceding terms in the sequence.
- **Sum of a finite geometric series**: The sum, S_n, of the first *n* terms of a geometric sequence is given by $S_n = \frac{a_1 - a_1 r^n}{1-r} = \frac{a_1(1-r^n)}{1-r}$, where a_1 is the first term and *r* is the common ratio ($r \neq 1$).
- **Sum of an infinite geometric series:** The general formula for the sum *S* of an infinite geometric series $a_1 + a_2 + a_3 + \ldots$ with common ratio *r* where $|r| < 1$ is $S = \frac{a_1}{1-r}$. If an

eries.html

- [ ] Purple Math: http://www.purplemath.com/modules/series5.htm

infinite geometric series has a sum, i.e. if $|r| < 1$, then the series is called a **convergent** geometric series. All other geometric (and arithmetic) series are **divergent**.

For further help:

http://intermath.coe.uga.edu/dictnary/homepg.asp
http://www.amathsdictionaryforkids.com/

Sample Practice Problems

1. The price of dance lesson depends upon the number of lessons that you select. If *x* is the number of lessons then the fee for the lessons (in dollars) can be found using the piecewise function
$$f(x) = \begin{cases} 40x \; if \; 0 < x \le 4 \\ 30x \; if \; 4 < x \le 8 \\ 25x \; if \; x > 8 \end{cases}$$
The lessons are increasing by 10% per lesson with a \$5 processing fee for each student. What is the new function for the cost of lessons?
$$f(x) = \begin{cases} 44x + 5 \; if \; 0 < x \le 4 \\ 33x + 5 \; if \; 4 < x \le 8 \\ 27.5x + 5 \; if \; x > 8 \end{cases}$$

2. Use the function $f(x)\sqrt[3]{5x}$ to answer the following questions:
What is the domain & range?
Domain: all real numbers; Range: all real numbers

What is the inverse of $f(x)$?
$f^{-1}(x) = \frac{x^3}{5}$

What is the domain and range of the inverse function?
Domain: all real numbers; Range: all real numbers

Over what line does the function and its inverse reflect across on the coordinate plane?
$y = x$

3. Identify the axis of symmetry, vertex, intercepts, domain, range, slope, and max/min of the following absolute value function:
$f(x) = |x| + 3$

A.O.S: $x = 0$; Vertex: (0, 3); x-intercept: none; y-intercept: 3; Domain: all real numbers; Range: $y \geq 3$; Left Slope: - 1; Right Slope: 1; Minimum: 3