Geometry: Unit 5
Geometric & Algebraic Connections

References

Textbook:

- HMH Analytic Geometry, Unit 6
- HMH Coordinate Algebra, Unit 6
- HMH Advanced Algebra, Unit 6

Online Access:
http://www.my.hrw.com

Helpful Links:

- Circle Equations: http://www.purplemath.com/modules/sqrcircle.htm
- Area and Perimeter on a Grid: https://mathbitsnotebook.com/Geometry/CoordinateGeometry/CGArea.html
- Distance Formula: https://mathbitsnotebook.com/Geometry/CoordinateGeometry/CGdistance.html
- Partition a Line Segment: https://mathbitsnotebook.com/Geometry/CoordinateGeometry/CGdirectedsegments.html
- Coordinate Geometry Proofs: https://mathbitsnotebook.com/Geometry/CoordinateGeometry/CGShowProofs.html

Dear Parents,

Students will use the concepts of distance, midpoint, and slope to verify algebraically geometric relationships of figures in the coordinate plane (triangles, quadrilaterals, and circles). Students will solve problems involving parallel and perpendicular lines, perimeters and areas of polygons, and the partitioning of a segment in a given ratio. Students will derive the equation of a circle and model real-world objects using geometric shapes and concepts.

Concepts Students will Use & Understand

- prove the slope relationship that exists between parallel lines and between perpendicular lines and then use those relationships to write the equations of lines
- extend the Pythagorean Theorem to the coordinate plane
- develop and use the formulas for the distance between two points and for finding the point that partitions a line segment in a given ratio
- revisit definitions of polygons while using slope and distance on the coordinate plane
- use coordinate algebra to determine perimeter and area of defined figures
- use Algebra to model Geometric ideas
- spend time developing equations from geometric definition of circles
- address equations in standard and general forms
- graph by hand and by using graphing technology
- develop the idea of algebraic proof in conjunction with writing formal geometric proofs

Vocabulary

- **Distance Formula:** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- **Formula for finding the point that partitions a directed segment AB at the ratio of *a* : *b* from A(x_1, y_1) to B(x_2, y_2):**

$$\left(x_1 + \frac{a}{a+b}(x_2 - x_1),\ \ y_1 + \frac{a}{a+b}(y_2 - y_1)\right)$$

or $$\left(\frac{a}{a+b}(x_2 - x_1) + x_1,\ \ \frac{a}{a+b}(y_2 - y_1) + y_1\right)$$

or $$\left(\frac{bx_1 + ax_2}{b+a},\ \frac{by_1 + ay_2}{b+a}\right)$$ ← **weighted average approach**

- **Center of a Circle**: The point inside the circle that is the same distance from all of the points on the circle.
- **Circle:** The set of all points in a plane that are the same distance, called the radius, from a given point, called the center. Standard form: $(x - h)^2 + (y - k)^2 = r^2$

- **Diameter**: The distance across a circle through its center. The line segment that includes the center and whose endpoints lie on the circle.
- **Pythagorean Theorem**: A theorem that states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.
- **Radius:** The distance from the center of a circle to any point on the circle. Also, the line segment that has the center of the circle as one endpoint and a point on the circle as the other endpoint.
- **Standard Form of a Circle:** $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the center and r is the radius.

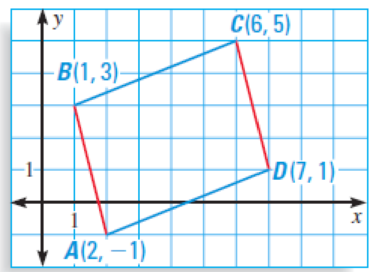
Sample Practice Problems

Example 1:

Write the standard form of the equation of a circle that passes through the given point (7,-4) and whose center is at the origin.

Example 2:

Show that $A(2, -1)$, $B(1, 3)$, $C(6, 5)$, and $D(7, 1)$ are the vertices of a parallelogram.



Key :

Example 1: $x^2+y^2=65$

Example 2:

SOLUTION

There are many ways to solve this problem.

Method 1 Show that opposite sides have the same slope, so they are parallel.

Slope of $\overline{AB}=\frac{3-(-1)}{1-2}=-4$

Slope of $\overline{CD}=\frac{1-5}{7-6}=-4$

Slope of $\overline{BC}=\frac{5-3}{6-1}=\frac{2}{5}$

Slope of $\overline{DA}=\frac{-1-1}{2-7}=\frac{2}{5}$

$\overline{AB}$ and $\overline{CD}$ have the same slope so they are parallel. Similarly, $\overline{BC}\parallel\overline{DA}$.

▶ Because opposite sides are parallel, $ABCD$ is a parallelogram.

Method 2 Show that opposite sides have the same length.

$AB=\sqrt{(1-2)^2+[3-(-1)]^2}=\sqrt{17}$

$CD=\sqrt{(7-6)^2+(1-5)^2}=\sqrt{17}$

$BC=\sqrt{(6-1)^2+(5-3)^2}=\sqrt{29}$

$DA=\sqrt{(2-7)^2+(-1-1)^2}=\sqrt{29}$

▶ $\overline{AB}\cong\overline{CD}$ and $\overline{BC}\cong\overline{DA}$. Because both pairs of opposite sides are congruent, $ABCD$ is a parallelogram.

Method 3 Show that one pair of opposite sides is congruent and parallel.

Find the slopes and lengths of $\overline{AB}$ and $\overline{CD}$ as shown in Methods 1 and 2.

Slope of $\overline{AB}$ = Slope of $\overline{CD}=-4$

$AB=CD=\sqrt{17}$

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel, so $ABCD$ is a parallelogram.